laws of springs are also relative to rest frame, (lab frame) when you do IGRP what's the difference with just the local rest frame of the system? What's the status of EGRP vs IGRP? Is one 'better'? (or 'more' relativistic?)

Relativity, Symmetries, and the Harmonies of Guitars [DRAFT]

Sebastián Murgueitio Ramírez

November 1st, 2023

Abstract

In this paper, I argue that there is an unnoticed ambiguity in standard presentations of the so-called Galilean principle of relativity, roughly, the principle that says the laws of mechanical systems are the same in all inertial systems. In particular, I argue that standard presentations fail to distinguish between what ultimately are two inequivalent principles, which I call "External Galilean Relativity Principle" (EGRP) and "Internal Galilean Relativity Principle" (IGRP). IGRP concerns the invariance of the laws of mechanical systems as seen from the perspective of a co-moving inertial frame. The paper is structured into three main parts. The first is mostly historical and shows that when physicists define and explain the Galilean principle of relativity, they sometimes refer to IGRP, some other times to EGRP, and sometimes to both. In the second part, I prove that IGRP and EGRP are not just two different versions of the very same principle but actually two inequivalent principles altogether. A direct and surprising implication of this result is that the laws of some (properly isolated) mechanical systems are not the same in all inertial systems. In the third part, I show how the distinction between IGRP and EGRP offers new insights into recent debates on the philosophy of symmetries.

Contents

1 Introduction

Imagine a guitarist (let's call her Alberta) playing in the restaurant cabin of a luxurious high-speed train. Because she was asked to do so by a girl in the audience, Alberta plays the very same piece when the train is parked at the station and, about an hour later, when the train is uniformly moving at 350 kph. To the trained ears of those in the audience (and Alberta herself), the tone of the chords is exactly the same in both cases. But why? A quick answer is that Alberta is using a good guitar that does not go out of tune in less than an hour. Sure, but what we are really asking is why is it that changes in the velocity of the train (and the guitar) with respect to the ground do not produce changes in the sound produced by the guitar as perceived in the cabin. A natural answer that comes to mind is that the waves associated with the guitar's strings are a classical phenomenon and thus satisfy the so-called "principle of Galilean relativity," roughly, the principle according to which the laws of mechanical systems are the same in all inertial frames. In slightly more detail, due to the principle of Galilean relativity, the laws for classical waves (such as the ones in the strings and also the sound waves propagating in the cabin) must remain invariant when we go from the inertial frame given by the cabin when parked to the inertial frame given by the cabin when moving at 350 kph (in slightly more technical jargon, these laws should remain invariant under Galilean boosts). Since these laws are the same in both frames, it follows that the vibrations in the strings are the same in both frames (when considering the same chords). Indeed, this is similar to what Einstein said when explaining a similar case:

> We should expect [if the principle of Galilean relativity is not true], for instance, that the note emitted by an organ-pipe placed with its axis parallel to the direction of travel [of Earth] would be different from that emitted if the axis of the pipe were placed

perpendicular to this direction [Einstein, 2015, p. 25-26].

So far so good, but there is a problem. Just as it happens with the case of electromagnetic waves, the laws governing acoustic waves and string waves are *not* invariant under Galilean boosts, and so these laws do not satisfy the Galilean principle of relativity. Hence, the explanation to our original question (and Einstein's explanation of the organ-pipe case) needs to be revisited, or so it would seem.

In this paper, I will argue that there is an unnoticed ambiguity in standard presentations of the Galilean principle of relativity. In particular, I argue that standard presentations fail to distinguish between what ultimately are two inequivalent principles of mechanics, which I call "External Galilean Relativity Principle" (EGRP) and "Internal Galilean Relativity Principle" (IGRP). I will show that these two principles are associated with two different ways in which the laws of mechanical systems can be said to be invariant under Galilean transformations, both of which play important roles in physics. Let me highlight two important implications of my view that will be developed in due time. First, there are cases where the laws of a certain system are not invariant under Galilean transformations, and yet such a system obeys one formulation of the Galilean relativity principle. Second, the ambiguity in the standard formulation of the relativity principle (which conflates two distinct principles) is very similar to one that affects various theses heavily discussed in the recent philosophical literature on symmetries. This suggests that the ambiguity in standard presentations of the Galilean principle of relativity noticed in this paper might ultimately be a result of a deeper ambiguity associated with standard definitions of physical symmetries more generally.

In section 2, I start with a brief historical overview of the Galilean principle of relativity and discuss some standard attempts at deriving the principle. Then, in section 3, I show that there are actually two different principles under the name “relativity principle,” namely, IGRP and EGRP. In section 4, I show that although various mechanical systems obey both principles, some do not. In section 5.1, I discuss some implications of this kind of framework regarding other recent debates in the philosophical literature on symmetries.

Before we dive in, let me note that everything I say here about the Galilean version of the relativity principle extends to the more general principle that Einstein introduces in his theory of special relativity (that one was supposed to include, in addition to mechanical systems, electromagnetic ones). But for reasons of space, and also to avoid unnecessary technicalities, I have decided to keep the discussion focused on the case of mechanics.

2 From Ships to Trains

In this section, we will go through some important moments in the history of the principle of relativity (restricted to mechanics). While the insights offered here might be familiar to historians of science, they serve as a foundation before we venture into the deeper philosophical implications of the principle. Let me add two warnings before we proceed. First, it is unclear that there is one single thesis (one single principle) picked out by the name "relativity principle." Second, not all physicists have regarded it to be a *principle* to begin (or regard *them* to be principles, if more than one). Instead, as we will see, many have thought of it (them) as a corollary that can be derived from other basic laws and principles. However, for the purpose of this section, I'll adhere to the traditional terminology used in history, philosophy, and physics, which is to use the phrase "principle of relativity." I urge readers not to be overly fixated on the specific wording or its singular implication. We'll circle back to these nuances in later sections.

2.1 Galileo

In Day Two of his *Dialogues*, Galileo (through Salviati) responded to various arguments aiming to defend the idea that the Earth does not move. The basic structure of these arguments was rather simple: If the Earth were in motion, this movement would manifest in observable effects on the behavior of objects on its surface, such as cannon balls and birds. Since we don't observe these effects, it then follows that the Earth remains stationary. Galileo's response centered on arguing that it was not true that the Earth's motion would produce any such effects, just as it is not true that the uniform motion of a ship produces any effects on the behavior of objects in the cabin of such a ship. When introducing what was going to become one of the most famous thought experiments in physics, Galileo said:

> Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water

> with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; [...] When you have observed all these things carefully [. . .], have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. [. . .] the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. [. . .] The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also.
> Galilei [1967]

It is easy to see why so many, including Einstein himself, attributed the principle of relativity (or a version of it) to Galileo. For it is very natural to read this passage as saying that the behavior of physical systems does not depend on the inertial frame used to describe them; the inertial frame associated with the ship at rest in the port, or the one given by the ship when sailing uniformly, lead to the exact same observations concerning the behavior of objects inside the cabin. But we should be careful when reading this passage from the lenses of a modern perspective given that for Galileo, motion with uniform speed around a *circular* path (not a straight one), such as the case of a ship navigating around the globe, would be an instance of inertial motion. Of course, there is not much harm for us in treating the ship as an (approximate) inertial frame because the curvature of the Earth is negligible given the relevant time and spatial scales involved in the experiments in the ship's cabin. And in any case, if we modify the example so that the ship is really following an inertial path as understood today, say if we consider a ship in outer space moving far away from any other bodies, then the passage in question would indeed illustrate the relativity principle as understood today. So, it is fair to say that this passage by Galileo does portray a very close cousin of the Galilean relativity principle as understood today.[1].

Another important point that we can conclude from the passage is that Galileo does not seem to treat his relativity principle (if we can call it that) as a principle but rather as some sort of consequence of other laws and principles (we will see that many scholars have followed Galileo in aiming to provide a derivation of the principle from other laws and principles).

[1] It is also useful to note that Galileo developed a theory of the tides that depended on the claim that certain states of circular motion around the Earth, such as the motion of the seas with respect to the Earth itself, do produce detectable effects. That theory seems to be in conflict with the relativity principle expressed in his passage (see Buchwald and Fox [2014, p. 793])

Galileo, in particular, explains why the systems inside the cabin behave the same way regardless of the cabin's uniform speed in this manner: "The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also." But what, exactly, does this mean? Galileo seems to be alluding to some combination of both his law of inertia together with his principle of the composition of motion. For example, think of the drops in the cabin that fall into the vessel beneath. As a drop is about to fall, it shares in the horizontal motion of the bottle, which is the motion of the ship. This is analogous to a cannonball inside a cannon pointed vertically; as the Earth rotates eastward, the ball inside moves eastward as well, even though it appears to remain stationary inside the cannon [Galilei, 1967, p. 176]. Once the drop leaves the bottle, it will "preserve" the same horizontal motion it already had simply because no other object or force interferes with its horizontal trajectory. Hence, some combination of the composition of motion and his law of inertia guarantees that "the ship's motion is common to all the things contained in it, and to the air also."

Unfortunately for Galileo, even if we were to replace his law of inertia with the Newtonian version (as presented in Newton's first law), his explanation would still be defective simply because many of the systems described in the cabin of the ship are not moving inertially (not even approximately). In the passage, we see references to things like the force required to throw something to your partner, the *effort* fish must make to swim, or the fact that butterflies don't get more tired if they fly in a direction opposite to the boat's movement direction. These examples are not instances of inertial systems because they appeal to forces of one kind or the other. Clearly, then, whatever the explanation of why these objects behave the same inside the cabin for two different states of motion of the ship is, it cannot be purely based on the law of inertia but it will require some dynamical considerations. As Brown [2005, p. 35] puts it "satisfaction of the relativity principle is not a mere consequence of the principle of inertia: the processes concern not just objects in motion but the dynamical mechanisms that produce that motion." This is precisely why, after Galileo, physicists have tried to explain the relativity principle, not through Newton's first law but through his second one (more on this soon).

2.2 Newton

The fifth corollary to the laws in Newton's *Principia* reads like this:

> Corollary V: When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly straight forward without circular motion [Newton, 1999, p. 423].

Notice that, unlike Galileo, Newton explicitly considers straight uniform motion. Also, note that by "space," Newton means a system that is approximately closed, for example, the cabin of a ship. Indeed, in the derivation of the corollary that he presents just afterward, he seems to allude to Galileo's ship: "This is proved clearly by experience: on a ship, all the motions are the same with respect to one another whether the ship is at rest or is moving" Newton [1999, p. 423]. Newton's derivation of this corollary appeals to both the second law and a situation involving objects colliding with one another, but it would be an understatement to say that the derivation is lacking. Indeed, as Brown 2005 has convincingly pointed out, it is not even valid. To see why, it might help to walk a bit slowly through an example similar to the one Newton describes just after presenting the corollary.

Let's consider the case where two billiard balls of the same mass, one blue and one yellow, collide head-on on a table inside the cabin of a ship initially at rest. Say that we track closely the motion of the balls with respect to the table from the moment they are 5 centimeters apart (before colliding) up to the moment they are 5 centimeters apart after the collision. Then, we repeat the experiment, with the same initial conditions (e.g., same relative velocities), but now the ship has some non-zero uniform velocity with respect to the shore. We will see, of course, that the balls behave in exactly the same manner in both experiments. Newton says:

> from these sums or differences [from their relative velocities] there arise the collisions and impulses with which the bodies strike one another. Therefore, by law 2, the effects of the collisions will be equal in both cases, and thus the motions with respect to one another in the one case will remain equal to the motions with respect to one another in the other case. [Newton, 1999, p. 423]

What Newton seems to be assuming is that the impulses solely depend on the relative velocities (which are the same in both cases). If the impulses are indeed the same, the accelerations produced during the collision will be the same when the ship is at rest and when it is moving uniformly. And once we have that the accelerations are the same, and have that the initial conditions are the same (they have the same initial velocity and position with respect to the table), it does indeed follow that their relative motions will be the same. The crucial premise, of course, is that the impulses are the same if the relative velocities are the same. In modern jargon, one can justify this premise in the following way: the force per unit of time that a ball feels during a collision equals the change in momentum of such a ball ($Fdt = dp$). And since the change in momentum only depends on the difference between the relative velocities of the balls and the masses (we are assuming elastic collisions), we can say that the force per unit of time only depends on such relative velocities and masses.

If all Newton wanted to demonstrate is that the behavior of objects that collide does not depend on the collective uniform velocity of the system (say the ship's velocity), then I think that this kind of derivation is enough. But, of course, Newton is trying to establish a very general result, one that is not restricted to one or other kind of force. Clearly, the prior derivation is far from establishing that very general result. This is precisely why Brown accuses Newton of committing a *non sequitur* in this derivation of corollary V. Brown says that "[...] significantly, Newton also presupposed the velocity independence of forces and masses" [2005, p. 37]. And a page later, he adds "The trouble is that the velocity-independence of forces and inertial masses is not a consequence of the laws of motion, as Barbour noted in 1989. Without this extra assumption, it is not possible to derive the relativity principle from Newton's laws and Galilean kinematics" [2005, p. 38]. To be fair to Newton, however, notice that he does not say that the independence of the (collision) forces is a consequence of the laws but only that the collision forces solely depend on the relative velocities of the bodies. So the *non sequitur* is not, contra Brown, that Newton took the non-dependence of forces on velocity to be a consequence of his laws but rather that he did not show nor explicitly assumed that *all* forces (not just collision forces) are independent of the velocities. Without explicitly adding this premise, one cannot reach Corollary V.

Curiously, recent physicists keep making a similar mistake when trying to derive the relativity principle. In section 10.2 of the first volume of his lectures, Richard Feynman says (my emphasis)

> In addition to the law of conservation of momentum, there is another interesting *consequence* of Newton's Second Law, to be *proved* later, but merely stated now. This principle is that the laws of physics will look the same whether we are standing still or moving with a uniform speed in a straight line. For example, a child bouncing a ball in an airplane finds that the ball bounces the same as though he were bouncing it on the ground. [. . .] This is the so-called *relativity principle*. Feynman et al. [1965, ch. 10].

Feynman is telling us that the principle of relativity (or the Galilean version, to be precise) is a consequence of Newton's second law and that he will in fact prove it later on. The alleged demonstration is presented in chapter 15 of the same volume. There, just after writing corollary V, Feynman say (emphasis in the original)

> This means, for example, that if a space ship is drifting along at a uniform speed, all experiments performed in the space ship and all the phenomena in the space ship will appear the same as if the ship were not moving, provided, of course, that one does not look outside. That is the meaning of the principle of relativity. This is a simple enough idea, and the only question is whether it is true that in all experiments performed inside a moving system the laws of physics will appear the same as they would if the system were standing still. Feynman et al. [1965, ch. 15]

Notice how Feynman is saying that all we need to answer now ("the only question") is whether this principle of relativity is true for all experiments. The implication, of course, is that he will go on to answer such a question. The way he aims to do that is by means of the Galilean transformations. In particular, he asks us to consider two inertial coordinate systems in relative motion with respect to one another. The position along the X axis in one system is related to the position in the other system via $x \mapsto x - vt$. Then, Feynman says this:

> If we substitute this transformation of coordinates [Galilean transformations] into Newton's laws we find that these laws transform to the same laws in the primed system; that is, the laws of Newton are of the same form in a moving system as in a stationary system, and therefore it is impossible to tell, by making mechan-

ical experiments, whether the system is moving or not. Feynman et al. [1965]

The problem, however, is that all the Galilean transformation on its own can show is that the acceleration is the same in both systems; $\frac{d^2}{dt^2}(x - vt) = \frac{d^2}{dt^2}x$. The transformation itself is silent regarding the force so Feynman cannot just use it to show that $F = ma$ is the same in both frames (the transformation is also silent regarding the mass, which must be assumed to be independent of the velocity as well). As happened to Newton, Feynman's derivation of the corollary is not complete unless we add the premise that all forces are independent of the absolute velocity of the system. And Feynman is not alone; see Susskind [2023, Ch. 1], Cline [2021, p. 9567], and Moataz [2021, p. 7] for some recent examples of physicists who have overlooked this important assumption.

what are some interesting forces which are velocity-dependent?

It is worth pointing out that the relativity principle is not just a principle about general laws such as $F = ma$ (although it includes them) but about specific laws such as, say, Newton's law of gravitation:

$$-G\frac{m_2}{|\mathbf{r_1} - \mathbf{r_2}|^3}(\mathbf{r_1} - \mathbf{r_2}) = \frac{d^2\mathbf{r_1}}{dt^2} \tag{1}$$

It is a simple exercise to show that such a law is indeed invariant under transformations of the form $\mathbf{r} \mapsto \mathbf{r} - \mathbf{v}t$. And notice that showing this already takes care of the velocity independence of the forces appearing in the law (we do not need to assume such independence, we can prove it from the invariance of the expression under boosts).[2] This highlights that one

[2]See Cartwright [1999, Ch. 3] for an insightful discussion regarding the nature of the

could try to derive a specific version of the relativity principle restricted to a particular force law without having to assume the velocity independence of the force in question (such independence is "built-in", so to speak, in the specific force law). Perhaps, then, a way to interpret Newton or Feynman is as implicitly adopting this kind of restricted version of the relativity principle, where the idea is not really to derive that the laws are the same in all inertial systems purely from $F = ma$ (which, again, does not work), but rather from the fact that all the specific (mechanical) force laws that we know of are, as a matter of fact, independent of the velocity (of course, the Lorentz force depends on the velocity, which is why we are restricted to purely mechanical systems). What if we take into consideration friction forces on mechanical systems, such as the force produced by air on an object falling from a building? Of course, that force does depend on the velocity between the object and the air, but in that case, we must treat the air and the object both as parts of an extended system. The total force on the extended system (a gravitational force in this case) remains independent of the velocity even though some of the internal forces depend on the velocities between the subsystems.

Before we move on, let's consider a question unaddressed in other discussions of the relativity principle. It may seem that any mechanical system, regardless of the specific forces involved, will satisfy Newton's second law (and a specific force law) in any inertial frame simply because that is what a mechanical system should do. If it were not to satisfy Newton's second law in all inertial frames, why would it be a mechanical system to begin

relationship between $F = ma$ and specific force laws.

with? Also, and related, if there was an inertial frame such that a certain mechanical system does not satisfy Newton's second law in it (or does not satisfy a specific force law in it), this would seem to imply that Newton's second law is not a law, to begin with (it would fail for some objects that are supposed to fall under its domain). But if we accept that a mechanical system must satisfy Newton's second law in all inertial frames simply as a matter of the definition of what mechanical systems are, then the relativity principle would seem to follow in a rather trivial manner. Call this the "triviality challenge," as it purports to suggest that the relativity principle is a trivial consequence of the fact that Newton's second law (and any specific force law) is indeed a law for mechanical systems. We will come back to this objection in later sections.

but why only inertial frames, and not all frames? and if okay to have limits, what's the worry?

2.3 Einstein

The reader might have noticed that our history of the relativity principle skipped over Einstein's own presentation, moving from Newton directly to Feynman's exposition. But this was on purpose. Einstein, in contrast to these other physicists, does not try to prove or derive the principle, and so I was trying to avoid discussing him in the same context. For Einstein, the relativity principle is a fundamental assumption that all mechanical and electromagnetic systems must adhere to. Using this, and assuming that the speed of light remains constant regardless of the source's speed, he goes on to prove that the relationship between time and length measurements in two different inertial frames cannot be described by the Galilean transformations but instead requires Lorentz transformations.[3]

As far as I can tell, the first use of the term "principle of relativity" appears in Einstein's 1905 paper on special relativity. There, he says that

> the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate Einstein [1905].

It thus becomes clear from the moment the term was coined that the principle of relativity ought to be treated as a postulate and not as a consequence of the laws. In section 2 of the same paper, Einstein defines the principle like this:

> The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion. Einstein [1905]

Some years later, in his popular book on relativity, Einstein introduced a restricted version of the principle, one that was supposed to only apply to classical mechanics. Then, he says that

[3] There is a debate regarding whether Einstein always viewed the principle of relativity as a postulate that leads to the transformation properties of the laws, or if he changed his mind later in life and believed that these transformation properties should be understood dynamically, through the dynamical laws of material bodies. Brown [2005] believed that Einstein later in life wanted to understand the Lorentz transformations dynamically, but Lange [2016, Ch. 3] disagrees with that view.

> If, relative to K [an inertial system], K' is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to K' according to exactly the same general laws as with respect to K. This statement is called the principle of relativity (in the restricted sense). As long as one was convinced that all natural phenomena were capable of representation with the help of classical mechanics, there was no need to doubt the validity of this principle of relativity. [Einstein, 2015, p. 24]

Notice the parenthetical remarks "in the restricted sense" precisely to indicate that this principle was supposed to be distinguished from the one that includes electromagnetism. Notice, also, that Einstein says that, as far as classical mechanics is concerned, there is no need to doubt the validity of the principle; all classical systems obey it.

3 Two different principles

3.1 Internal vs External

Recall that Einstein characterizes the principle as the claim that (classical) natural phenomena run their course with respect to K' according to exactly the same laws as with respect to K. But what does this mean, exactly? Presumably, it means that if we describe the behavior of a physical system S using frame K, we arrive at the same law for that system as the law we would have arrived at had we used frame K'. But this is still not completely clear.

Are we considering a single mechanical system as seen from the perspective of two different frames in relative motion with respect to one another, or are we considering two copies or instances of the same type of system, where one copy is co-moving with frame K and the other copy co-moving with frame K'? To better understand this question, consider a very simple example.

Say that Sara, inside a train, will throw a dart at a board hanging in the front wall of the cabin (so the throw is in the direction of motion of the train). In the cabin, we have installed a camera that records the motion of the objects and is also capable of making measurements of speed, time, and acceleration. Imagine, furthermore, that the cabin of the train is made out of glass (so much for privacy), so observers outside can see what is happening in the cabin. We also have installed a second camera outside, next to a light post. When the cabin is passing by the light post with a constant speed of 100 kph, Sara throws the dart. We then collect the movies from the two cameras and compare them. As expected, both recordings indicate the same vertical acceleration (g), no horizontal acceleration, and the same time for the dart's flight. They also indicate different measurements for the traveled distance and the velocity of the dart, as one camera sees things from outside while the other one is moving together with the cabin. Crucially, both sets of measurements are consistent with Newton's second law. In particular, they both recover that the dart's weight equals the mass times the vertical acceleration ($mg = ma$) and that zero horizontal force equals zero horizontal acceleration ($0 = m0$). One can then say that the dart obeys $F = ma$ in the frames of the two cameras (in physics jargon, $F = ma$ remains invariant under boosts). This is then a sense in which the same phenomena (the dart's behavior) "run their course with respect to K' according to exactly the same **general** laws as with respect to K."

We again have Sara throwing a dart inside a train's cabin. The difference now is that instead of a single throw, we will consider two different throws, and we will forget about the external camera. First, when the train is parked at the station, Sara will throw the dart, and the camera in the cabin will record the motion. Then, when the train is moving at 100 kph, Sara will throw the dart again, with the same initial speed (with respect to her) and at the same angle. We then collect the movies from the two throws and compare them. As in the other case, both recordings indicate the same vertical acceleration (g), no horizontal acceleration, and the same time for the flight. Thus, if one recording shows that the dart satisfies $g = a$, the other one will also show this. But in contrast to the previous case, now the two recordings also show the same distance, the same velocity, and, in general, the very same motion. In fact, we could not tell one movie apart from the other one! We then say that both throws of the dart (each happening at a different speed with respect to the ground) obey the very same general law. This is then a second sense in which the same phenomena "run their course with respect to K' according to exactly the same **general** laws as with respect to K."

There are, then, at least two different senses for the phrase "obeying the same laws in different inertial frames" and so two readings of the relativity principle (or two different relativity principles altogether). In one sense, that I call "external," the phrase means obeying the same laws for a given *single* system simultaneously studied from two (or more) inertial frames. In a second sense, that I call "internal," that phrase means that if we study a system from the perspective of a certain inertial frame, and we were to boost both the frame and the system together with respect to the first frame, then the laws of the boosted system from the perspective of the boosted frame are the same as the laws of the original system from the perspective of the original frame (notice that this assumes that the initial conditions, as seen from the perspective of the frame in which the experiment is conducted, are the same). It is important to note that when considering the internal sense, we don't necessarily need to think of a single system that is boosted. Instead, we can also examine two instances of the same type of system, such as two darts from the same batch. For instance, let's say that at 3:00 pm, Sara throws a dart with initial conditions C while the train is moving at 100 kph, and at the same time, Carlos throws a dart of the same kind with the same initial conditions C inside a train moving at 5 kph. According to the internal interpretation of the phrase in question, both Carlos and Sara will observe their darts following the same laws of motion, and even the exact same trajectory. In short, we arrive at these two principles:

> **External Galilean Relativity Principle (EGRP)**: A given mechanical system S_1 behaves in an inertial frame K according to exactly the same laws as the ones it obeys in an inertial frame K' in uniform motion with respect to K.

> **Internal Relativity Principle (IRP)**: Take a given mechanical system S_1 of type T in an inertial frame K co-moving with it. Take a second mechanical system S_2 of type T uniformly moving

> with respect to S_1, and consider an inertial frame K' co-moving with S_2. The laws that S_1 obeys according to K are exactly the same laws as the ones S_2 obeys according to K'.[4]

We might think that it is rather obvious that the relativity principle is the one corresponding to the internal version (IGRP) in this classification. IGRP does seem to be the one alluded to in Galileo's passage (with the caveats mentioned then), the one mentioned by Newton in Corollary V, and the one Feynman presented with the spaceship example discussed earlier. IGRP is also the one presented by philosophers of physics such as Brown 2005 and Norton 2008. However, not all physicists agree. For example, Moataz [2021, p. 105] defines the Galilean relativity principle in the following way: "Any two observers moving at constant speed and direction with respect to one another will obtain the same results for all physical experiments," and then goes on to clarify that *the observers will disagree about properties such as the velocity*, which can only happen if one is thinking of EGRP [2021, p. 106]. To give another example, consider what Wheeler says here, when illustrating the principle of what he calls "Galilean relativity":

> Relative uniform motion of the two ships does not affect the laws of motion in either ship. A ball falling straight down onto one ship appears from the other ship to follow a parabolic course; a ball falling straight down onto that second ship also appears

[4] It is worth pointing out that when saying that S_2 is in uniform motion with respect to S_1, this does not exclude the case in which S_2 and S_1 both have some acceleration in their corresponding frames. Rather, this just means that for any given instant of time t, the velocity at that instant in one frame is related to the velocity in the other frame by a constant factor.

> to follow a parabolic course when observed from the first ship. [Taylor and Wheeler, 1992, p. 57713]

And finally, consider what Hughes says here:

> This [relativity principle] does not mean that observers in all inertial frames agree on the specific value of an object's momentum; indeed, you should be able to convince yourself that you can make that object's momentum take any value at all by changing frames of reference. However, all observers agree that if that body interacts with another body, then the momenta of the two bodies after their interaction is the same as it was before. [Hughes, nd, p. 7]

Notice that in all these passages, the physicists point out that the actual trajectories of the bodies will look different in the different frames, which only happens in the external interpretation of the relativity principle. Thus, some physicists do indeed explicitly define the relativity principle through the lenses of EGRP.

The natural question to consider now is what the connection between the internal and the external interpretation exactly is. A clue towards the answer lies in the attempted derivation by Feynman that we saw earlier. If we look carefully, all the derivation recovers, once we show or prove that the force is independent of the velocity, is EGRP. In particular, notice that throughout the derivation there was a single system, and the laws for that system were being studied from the perspective of two different inertial frames. The conclusion of the derivation is supposed to be that the form of the laws

($F = ma$ or the law of universal gravitation), as seen in a certain inertial frame, is the same as the form of the laws as seen in a different inertial frame related to the one by a Galilean boost (i.e., $x \mapsto x + vt$). But if this is all the derivation achieves, notice that it does not show what would have happened if the same system were to be boosted together with the frame, which is what we need to get IGRP. Hence, Feynman and others mistakenly think that they have derived IGRP (which is the principle Feynman illustrates with the spaceship example) just from the invariance under Galilean transformations.

Say, then, that we have successfully derived EGRP along the lines discussed in the previous section (which includes assuming or proving that the force is independent of the velocity). How can we go now from EGRP to IGRP? It turns out that this is not hard given the assumption that the force is independent of the velocity. Say that we have already shown that the system obeys the same laws in two inertial frames related by a Galilean boost, say R_1 and R_2 (this was the point of the previous derivation). Crucially, this result should be preserved even if the system (or a copy of it) is boosted because (a) the forces and masses are assumed to not depend on the velocity and (b) the accelerations are invariant under Galilean transformations (as Feynman shows). Hence, consider a boost of the system of the very same magnitude and direction as the boost that takes us from frame R_1 to frame R_2. The boosted system will obey the same laws in both systems (this follows from the derivation of IGRP), but in addition to this, the *relative motion* of the boosted system with respect to R_2 will be just the same as the relative motion of the non-boosted system with respect to R_1 (if we boost the system together with the observer, then the relative motion of the system with respect to the observer will be preserved). In short, then, we can move from EGRP to the IGRP simply by boosting the system with the same velocity that maps a frame R_1 into a frame R_2. It seems, then, that the same assumptions appealed to when deriving EGRP can take us all the way to IGRP. Although tightly related, the principles are logically independent from one another. In particular, one cannot derive EGRP from IGRP (although one can derive IGRP from EGRP, as we just did). This will be clear in the next section.

4 Explanation and invariance

Go back to the example of Sara throwing a dart inside the train. Why is it that the dart behaves in the same way when the train is parked, and when it has uniform motion? We can answer, of course, that this is a consequence of the relativity principle. But suppose we believe that this kind of explanation is dubious (perhaps we regard the relativity principle to be a consequence of the laws, or we simply think that the question is seeking a causal answer). Suppose, in particular, that we want to give an answer that directly appeals to the dynamical laws of the dart. How can we answer in that case? A standard approach consists of showing that the laws for the dart are invariant under boosts. In particular, as it flies, the dart satisfies the equation $\frac{d^2}{dt^2}y = g$ along the vertical axis (constant acceleration), and $\frac{d^2}{dt^2}x = 0$ in the horizontal axis (no acceleration). It becomes an elementary mathematical exercise to show that neither of these equations changes under the boost transformation $x \mapsto x - vt$. Mathematically, this shows that the transformation is a symmetry of these differential equations. Physically, this is taken to represent the fact that the dart will behave in the same way regardless of any changes in its horizontal velocity (or, if one interprets the boost passively, regardless of changes in the uniform motion of the frame along the horizontal direction). So the invariance of the dart's dynamical laws under boosts explains why the dart behaves the same way when the train is stationary and when it is moving uniformly. Similarly, the invariance of the dart's dynamical laws under spatial shifts, such as $x \mapsto x - d$ (with d constant) explains why the dart behaves the same way when the train is parked at the first station, and when it is parked in the second one. And, more generally, the invariance of the dart's dynamical laws under a transformation T explains why the dart's behavior is the same when its state is transformed according to T (or, in the passive case, when the frame itself is transformed by T). The idea that dynamical symmetries (transformations that preserve the form of the dynamical equations) play a crucial role in these kinds of explanations seems to be central in both the physics and philosophical literature (e.g., Wallace [2022]).

Consider, now, a more complex mechanical system such as the vibration of the strings of a guitar that Sara is playing on the train. The dynamical equation for the waves in the strings is

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}, \tag{2}$$

where T is the tension, μ the density in the string, t time x the horizontal displacement and y the vertical displacement. Why is it that the waves behave in the same way (e.g., form the same oscillations, produce the same sounds, etc) when the guitar is played at the train's first station and when played at the second one? Well, we can proceed as we did before, namely, we can check that the equation is invariant under $x \mapsto x - d$ (to do so, we need to use the chain rule in order to show that the partial derivative operator with respect to x does not change, that is, $\frac{\partial}{\partial x} = \frac{\partial}{\partial x'}$). So far so good. But now ask: Why is it that the waves behave in the same way when the guitar is played at the train's first station and when played as the train is moving with uniform speed with respect to the station? Once again, we can try to check that $x \mapsto x - vt$ leaves the equation invariant, but it turns out that this is not true. Instead, if we were to apply such transformation, the equation becomes:

$$\frac{\partial^2 y}{\partial t'^2} + 2v \frac{\partial^2 y}{\partial x' \partial t'} + \left(v^2 - \frac{T}{\mu}\right) \frac{\partial^2 y}{\partial x'^2} = 0. \tag{3}$$

The solutions to this equation are much more complex than the ones for equation 2 (unless $v = 0$, in which case the equations are the same). But we do not need to explicitly discuss the solutions here. The important point is that, unlike the equations for the dart, the one for a classical wave is not invariant under Galilean boosts.

Of course, it is well-known that the failure of invariance of electromagnetic waves under Galilean boosts was a major concern in physics during the second part of the 19th century, and a major source of inspiration for Einstein's development of special relativity (see Buchwald and Fox [2014, ch. 26]). Here, however, we are talking of a classical system, a simple string attached to two fixed points on a guitar. Isn't it precisely a signature of classical systems that they remain invariant under Galilean transformations? At this point, the reader might be thinking something along the following lines: the lack of invariance, in this case, is simply a consequence of the fact that the equation assumes a certain frame, namely, a frame in which the strings remain at rest when not vibrating. A similar thing happens with sound waves; the equation in that case assumes a frame in which the air molecules are initially at rest, and the waves are modeled precisely with respect to that frame. If we were to boost the object emitting the sound with respect to that rest frame, or if we were to boost the air molecules (say with a sophisticated air tunnel), then the sound emitted by an object would no longer satisfy the standard wave equation but a more complex one. Indeed, this is precisely the same reason that led many physicists, including Maxwell himself, to posit that electromagnetic waves ought to be understood with respect to a special medium such as the ether. This is why many believed that if the Earth was moving with respect to such a medium, we would have expected the light to behave a bit differently as the Earth was orbiting the Sun.

The previous explanation amounts to this: the failure of equation 2 to remain invariant under Galilean boosts is a sign that the equation implicitly (or explicitly) presupposes a specific "rest frame," namely, the frame in which the medium of propagation for the wave is at rest. I agree with that diagnosis. However, I believe that its implications have not been fully appreciated in the literature for at least five reasons.

First, the lack of invariance under boosts for mechanical waves illustrates that not all inertial systems are equivalent when it comes to the description of mechanical systems, contrary to what is usually said in textbooks and classes. When objects are in free fall, sliding over a surface, or interacting gravitationally, the choice of an inertial system does not affect the form nor the simplicity of the law-equations. But a frame that is initially at rest with respect to the system producing mechanical waves does produce a simpler law compared to the law that would be obtained if we were to choose a frame that is moving relative to the medium (i.e., the equations for waves in a frame in motion with respect to the medium are much more difficult to solve). And this point is not just restricted to waves in strings but is a very generic feature of mechanical systems whose equations involve certain kinds of partial differential equations; the propagation of heat on a rod, the oscillation of a spring around an equilibrium point, and even the motion of a body in a central potential (where the potential is considered to be at rest in the origin). In all of these cases, the specific laws for the systems involve differential equations that fail to be invariant under boost transformations (Belot [2013] offers a detailed discussion of such cases). For example, it is easy to show that $x = -\omega^2 x''$, the equation of a classical harmonic oscillator, is not invariant under $x \mapsto x + vt$. This entails that the motion of a spring with respect to a moving frame is no longer described by a purely harmonic function, such as $x(t) = A\cos(\omega t)$, but rather involves additional linear terms of the form vt. The reason is that the boost of the frame (or of the spring itself, in the active case) makes the spring no longer oscillate around the same equilibrium point, which is precisely what we take springs to do (for a detailed discussion of this case, see Murgueitio Ramírez [2022]). The world of mechanical systems is full of examples of phenomena whose specific laws privilege the use of some inertial systems above others, and so it is full of systems whose specific laws are not manifestly Galilean invariant (even though they are typically invariant under purely spatial translations).

Second, and related to the prior point, there can be cases in which the more general laws for a system are invariant under boosts even though the specific laws are not. For example, by dividing (in our minds!) the string into very small segments, one can derive the wave equation for the string by using both $F = ma$ and the assumption that the tension is constant throughout the string. Crucially, such tension does not depend on the velocity of the string itself with respect to an external system (otherwise we would have to keep into account the velocity of the Earth when modeling drum waves in an acoustic lab!). Hence, neither the forces on the string nor the string's acceleration depends on the string's velocity with respect to an external system. Yet the specific law equation at which we arrive does depend on such a velocity, as the difference between equation 2 and equation 3 highlights. This is significant because it shows that two observers in different states of relative motion will agree about the relationship between the tension on a segment in the string and the acceleration of that segment, but disagree about the way such a segment actually behaves (they will use different differential equations for the motion of the same segment). This is why, when discussing the relativity principle, one must be careful to distinguish between very general laws such as $F = ma$ and the more specific laws that can be derived from these.

Third, the failure of invariance for some mechanical systems under boosts entails that those systems do not satisfy EGRP; their specific laws, when seen from different inertial frames, are not the same. For instance, as the (transparent!) train passes by, a camera in the train station will see the vibrations in Sara's guitar to obey a different differential equation than the camera inside the train's cabin. Similarly, had Sara hung a pendulum from the roof of the train cabin, the camera inside would have seen the mass of the pendulum behave according to a different differential equation than the camera outside (and this would not have happened had we considered a pure spatial translation). Notice that these are not simply cases where the particular motion of the object looks different, such as differences in the horizontal velocity of the dart's motion when comparing the movie of the external camera with the movie of the internal one. Rather, these are cases where the dynamical laws of the systems themselves (as represented by the differential equations physicists use to model these systems) are different. And so these are cases where EGRP is not satisfied.

Fourth, notice that mechanical systems can satisfy IGRP even if they do not satisfy EGRP, which means that one cannot derive EGRP from IGRP (EGRP is stronger). The camera inside the cabin will see the waves in the guitar strings look just the same when the train is parked and when it is moving. And so, in particular, the specific laws that we can derive from the movie recorded by such a camera will be exactly the same in the parked train and the moving train (we will recover the same differential equations in this case). This is just a different instance of the fact that we can use the same wave equation to model all kinds of mechanical waves even though the Earth has different velocities as it orbits the Sun. All this is to say that the string does satisfy IGRP even though it fails to satisfy EGRP. This is a way of proving that these principles are not equivalent, as the satisfaction of IGRP does not guarantee the satisfaction of EGRP.

Fifth, and finally, if a system can satisfy IGRP without satisfying EGRP, it follows that there is a deep problem with the kinds of explanations that we encountered at the beginning of the current section. Those explanations appealed to the invariance of the dynamical laws to answer why the system behaves in the same way after various transformations. Indeed, this is exactly the explanation we gave for why spatial translations of the wave do not change the wave's behavior (recall that we appealed to the fact that the wave equation is invariant under $x \mapsto x - d$ in order to explain this). However, the wave equation does not remain unchanged under Galilean boosts, despite the fact that these waves fulfill IGRP. This takes us back to the initial question that we raised at the start of this essay: If the classical wave equation is not invariant under constant changes in velocity, what explains why the guitar produces the same sounds when the train is parked and when it is moving at 350 kph? Why, in other words, does the guitar obey IGRP if it does not obey EGRP? This is the question that I want to answer in the next section.

5 Some implications of the EGRP vs IGRP distinction

5.1 Internal vs External Laws

I believe that the key to explaining why the string waves behave in the same way inside the cabin when the train is boosted is actually simple; when we boost a system together with an observer, then the motion of the system with respect to the observer is preserved. In more detail, say that $x_1(t)$ is the motion of a system with respect to a frame R_1, and $x_2(t)$ is the motion of the observer (it can be a detector or an agent) with respect to that same frame. The behavior of the system with respect to the observer will be captured by the relative motion of the two objects, that is, by $z(t) = x_1(t) - x_2(t)$. Then, say that we boost the frame. Such a boost will change the position coordinates of all objects through $r \mapsto r + vt$. However, the relative motions will be preserved: $z'(t) = (x_1(t) + vt) - (x_2(t) + vt) = x_1(t) - x_2(t)$. Crucially, the exact same reasoning would work in the active case, except that now we imagine the boost as giving the systems (not the frame) an additional velocity of vt. That is, the boost of the systems will preserve the relative motion among such systems just as a boost of the frame does. The preservation of that relative motion (as a function of time) is all that is required to recover the kind of fact that we wanted to explain, namely, the fact that boosting the train does not change how the string behaves inside the cabin.

Notice that in the prior explanation, we did not say anything about the dynamical laws of the target system or the observer. The laws, of course, allow us to recover the particular motions, but once we have these motions we do not need to go back to consider the laws. We also do not need to consider whether the dynamical laws themselves are invariant or not under boosts. As the case of the waves illustrates, the invariance of the system's laws with respect to an external system is not essential to the explanation in question. We actually did not even need to appeal to waves to make this point. Take, once again, the dart. Why is it that its motion is the same with respect to Sara when the train moves and when it is parked? The answer is not, as we said before, that the laws for the dart have certain symmetries (such as invariance under spatial translation). Rather, the answer is much simpler; the relative motion of the dart with respect to Sara, as given by $z(t) = x_1(t) - x_2(t)$, is preserved by boosts. The answer, in particular, does not need to appeal to the invariance of the differential equations for the dart's laws. Notice, for example, that if we were to consider an acceleration of the frame in a straight line, such an acceleration would not preserve the dynamical laws for the dart. In such a case, we would have to add a fictional force to account for the apparent acceleration of the dart (and Sara) with respect to the accelerating frame. In other words, the dart's laws are not invariant under constant accelerations. And yet, note that such an acceleration would also preserve $z(t) = x_1(t) - x_2(t)$ because the very same acceleration affects both Sara and the dart (as seen from the frame) and so it will be canceled out when subtracting the position of both. The same is true in the active case, as long as the accelerations are the same for all objects (as in the case of a uniform gravitational field). So, once again, this simple example illustrates that the dynamical laws of a system might fail to be invariant under a certain transformation (a boost, or a constant acceleration) even though such a transformation preserves all of the internal relative motions.

The previous discussion thus motivates a distinction between "internal laws" and the "external laws." The former concerns the behavior of the system as seen from the perspective of a co-moving reference system (which is transformed together with the target system), such as Sara in the cabin. The latter concerns the behavior of the system as seen from the perspective of an external frame, such as someone standing in the station (which is not transformed together with the target system). Using this terminology, we can then say that a given transformation might be a symmetry of the internal laws but fail to be a symmetry of the "external laws," as boosts of the wave illustrate. Notice that if a certain transformation is a symmetry of the external laws for a system, then the system satisfies $EGRP$. And if a certain transformation is a symmetry of the internal laws, then the system satisfies $IGRP$. One and the same transformation can be a symmetry in the internal sense and yet fail to be a symmetry in the external sense. This is another way of showing that a system can satisfy $IGRP$ even when it does not satisfy $EGRP$, and so a way of showing that the explanation of why a system satisfies $IGRP$ should not depend on $EGRP$. The explanation, rather, seems to appeal to the simple fact that transformations of all the objects in a certain enclosed system along a straight line will preserve the relative degrees of freedom of the various subsystems.

5.2 Conserved quantities and the external perspective

As explained above, one must adopt something like the internal perspective if we want to understand why boosting certain mechanical systems does not affect their behavior as seen from a frame that is also boosted. For some explanations, however, one must adopt the external perspective. In particular, the external perspective is more adequate when considering certain physically significant facts about symmetries. For example, consider one of the most important facts that we learn in introductory physics classes about symmetries; that, due to Noether's first theorem, there is an interesting connection between symmetries and conserved quantities. For instance, if a system remains invariant under constant spatial translations, then its linear momentum remains conserved. Say that we have a detector inside a spaceship that measures the linear momentum of objects, and we shift the spaceship by a constant amount with respect to an external system. If we repeat the same experiments with the same initial conditions, the detector will indicate the same readings for the momentum of objects inside the ship.

relation to passive / active symmetries?

This example (which describes things from an internal perspective) appears to show that linear momentum is conserved by spatial translations. However, it is a misleading demonstration. If we were to boost the spaceship (and all items inside), and use the same device (also boosted), we would still obtain the same linear momentum measurements for all objects within the cabin. Despite this, linear momentum is *not* the quantity that is conserved by boost transformations according to Noether's first theorem. Spatial translations, boosts, and any non-rotational transformation that acts on all the objects in the same way (including the measurement device) will preserve the linear momentum readings inside the ship.[5] Hence, the internal perspective (in which both the observer and the system are transformed together) obscures the connection between the conservation of certain quantities and symmetry transformations. From the internal perspective, it seems as if the conservation of linear momentum was trivial, easily obtained from all sorts of transformations!

The lessons of Noether's first theorem are recovered, not by the internal perspective but by the external one. The theorem does not entail that the momentum measured inside the ship at rest and in uniform motion is the same, given the same system and initial conditions. Rather, it entails that two external observers looking at the same system from different locations will agree on the linear momentum of the objects. If one observer is moving faster than the other, they will disagree on the velocities and momentum of the system (i.e., the momentum is not conserved in such a case). This is precisely why there is an interesting connection between spatial shifts and linear momentum and not between boosts and linear momentum. In the case of boosted frames, Noether's first theorem entails that the observers will agree, not on the value of the linear momentum, but on the value of the quantity $x_{CM} - v_{CM}t$, where x_{CM} is the center of mass of the system, v the velocity associated with the boost and t time. This property may not be immediately intuitive (it has to do with the uniform motion of the center of mass of an isolated system), but it is conserved and simply not measurable

[5] This includes time-dependent accelerations, as long as they act on all the bodies in the same way, as it happens in a sufficiently uniform gravitational field. This can be thought of as a consequence of Corollary VI Newton [1999].

from an internal perspective.

5.3 Connection to other debates

In recent years, philosophers have tended to emphasize that symmetries preserve appearances (i.e., Ismael and van Fraassen [2003], Saunders [2003], Baker [2010], Dasgupta [2016]). As far as I can tell, they seem to be adopting an internal perspective, as they are usually considering cases where everything, the system, and the observer included, are transformed in some way (as in the Leibniz Clark correspondence, where we shift all the matter certain distance in a given direction). However, as the simple example of the spaceship shows, it is often the case that physics practice requires the adoption of an external perspective. In such cases, only certain aspects of the "appearances" are preserved, as a symmetry transformation will disrupt observable properties such as the distance or the velocity between the system and the external observer. Furthermore, as the case of the waves in the string illustrates, some transformations such as boosts of the frame do not preserve the form of the law of certain mechanical systems, and so do not count as dynamical symmetries of such systems. And yet, if we were to consider those transformations in conjunction with a boost of the observer or frame, we would get the result that the internal appearances are indeed preserved (i.e., IGRP would be satisfied even if EGRP would not). This suggests that the link between symmetries and appearances is weaker than what has been suggested in the recent philosophical literature because non-symmetry transformations can also preserve appearances (for further discussion, see Murgueitio Ramírez [2022]).

For similar reasons as with the case of the relationship between symmetries and appearances, I think that another recent debate on the philosophy of symmetries has conflated external laws and internal ones. For example, Belot [2013] has argued that many symmetry transformations do not preserve the physical state of a system, including symmetry transformations of mechanical waves (up until this point we showed something else, namely, that boosts are not symmetries of mechanical waves). I believe that many of the transformations that Belot discusses ought to be understood externally, that is, as changing the external physical state (the state as seen from an external frame). The examples are certainly interesting, but they do not suffice to establish that the states of the corresponding systems do change if one were to transform both the system and the frame (as in the case where we transform the wave and the frame). This highlights that there might be a way of (but it is not guaranteed) reconciling these examples with the internal perspective, which is the one I think many philosophers adopt when thinking of symmetries as preserving physical states.

References

Baker, D. J. (2010). Symmetry and the metaphysics of physics. *Philosophy Compass*, 5(12):1157–1166.

Belot, G. (2013). *Symmetry and Equivalence*, pages 318–339. Oxford University Press.

Brown, H. R. (2005). *Physical Relativity: Space-time structure from a dy-*

namical perspective. Oxford University Press UK, Oxford.

Buchwald, J. Z. and Fox, R., editors (2014). *The History of Physics*. Oxford University Press.

Cartwright, N. (1999). *The Dappled World: A Study of the Boundaries of Science*. Cambridge University Press.

Cline, D. (2021). Inertial frames of reference. LibreTexts Physics. Retrieved on August 29, 2023.

Dasgupta, S. (2016). Symmetry as an epistemic notion (twice over). *The British Journal for the Philosophy of Science*, 67(3):837–878.

Einstein, A. (1905). On the electrodynamics of moving bodies. In *The Principle of Relativity*, pages 35–65. Dover Publications.

Einstein, A. (2015). *Relativity, the Special and the General Theory: 100th Anniversary Edition, with Commentaries and Background Material by Hanoch Gutfreund and Jurgen Renn*. Princeton University Press.

Emam, M. H. (2021). *Covariant Physics: From Classical Mechanics to General Relativity and Beyond*. Oxford University Press.

Feynman, R. P., Leighton, R., Sands, M., and Hafner, E. (1965). The feynman lectures on physics; vol. i. *American Journal of Physics*, 33:750.

Galilei, G. (1967). *Dialogue concerning the two chief world systems, Ptolemaic and Copernican*. Modern Library Classics. University of California Press, Berkeley, 2d. ed. edition.

Hughes, S. A. (n.d.). Lecture 1: Introduction to classical mechanics. `https://web.mit.edu/sahughes/www/8.033/lec01.pdf`. PDF document.

Ismael, J. and van Fraassen, B. C. (2003). *Symmetry as a Guide to Superfluous Theoretical Structure*, pages 371–392. Cambridge University Press.

Lange, M. (2016). *Because Without Cause: Non-Causal Explanations in Science and Mathematics*. Oxford University Press.

Murgueitio Ramírez, S. (2022). Three puzzles about symmetries. In *Philosophy of Science Association*, Pittsburgh. PSA.

Newton, I. (1999). *The Principia: Mathematical Principles of Natural Philosophy*. The Principia: Mathematical Principles of Natural Philosophy. University of California Press, Berkeley.

Norton, J. D. (2008). Why constructive relativity fails. *The British Journal for the Philosophy of Science*, 59(4):821–834.

Saunders, S. (2003). Physics and leibniz's principles. In Brading, K. and Castellani, E., editors, *Symmetries in Physics: Philosophical Reflections*, pages 289–307. Cambridge University Press.

Susskind, L. and Cabannes, A. (2023). *General Relativity: The Theoretical Minimum*. The Theoretical Minimum. Basic Books.

Taylor, E. F. and Wheeler, J. A. (1992). *Spacetime Physics*. LibreTexts.

Wallace, D. (2022). Observability, redundancy and modality for dynamical symmetry transformations. In Read, J. and Teh, N., editors, *The*

Philosophy and Physics of Noether's theorem, pages 322–353. Cambridge University Press, Cambridge.