Superfluous Ideological Superstructure (or: What Humeans Should Say about Quantum Mechanics, and what *that* Says about Humeanism)

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1 Introduction

Distilled to its most essential elements, textbook quantum mechanics gives us a piecewise algorithm for predicting the behavior of any quantum system:

$$\begin{cases} \text{Schrödinger equation} & \text{if system is not measured} \\ \begin{array}{l}\text{Measurement postulate} \\ \text{(Born rule + collapse} \\ \text{ onto eigenstate of } A\text{)}\end{array} & \text{if system variable } A \text{ is measured} \end{cases}$$

When the system is not measured, we can predict its temporal evolution by figuring out its Hamiltonian and current state, plugging these into the Schrödinger equation, and solving for its state at other times. When the system is measured, then we apply the measurement postulate, itself a combination of two rules: the Born rule gives us the probabilities of the different possible measurement outcomes, and the collapse postulate tells us that if we get a particular outcome, then the state of the system "collapses" to one in which that outcome is guaranteed if we immediately measure it again.

Most physicists will tell you that textbook quantum mechanics[1] is perfectly fine for all practical purposes—i.e. in all realistic situations where we want to predict the behavior of some quantum system, textbook quantum mechanics is predictively adequate. Indeed, the physicist John Stewart Bell once coined the acronym *FAPP*, "for all practical purposes," for exactly this—ah—purpose (Bell, 1990).[2]

Nevertheless, many philosophers of physics will tell you that textbook quantum mechanics is inadequate as a fundamental theory of the world. And of

[1] I'll speak here as if "textbook quantum mechanics" is a monolithic entity, though of course there are differences in its presentation in different sources. Some sample textbooks that I have in mind are Dirac (1981), Landau and Lifschitz (1977), and von Neumann (2018).

[2] See also Maudlin (2019: 3-4).

course it *is*, for straightforward physical reasons: there are difficulties getting it to cohere with general relativity. But even setting conflicts with relativity aside (as I will here), the usual concerns are rather more philosophical. There are two of them.

First is the measurement problem. Here the thought is that the piecewise dynamical algorithm that we saw above cannot possibly give us the fundamental quantum mechanical laws of nature. That's because if we try to understand the algorithm in this way, it will imply that the fundamental laws must be paying attention to whether or not physical systems are being measured, and directing their behavior accordingly. But "measurement" is an anthropocentric and vague concept that reflects various contingent humans interests and concerns; it is not the kind of thing that could plausibly figure into the fundamental laws. In order to have a realistic proposal for the fundamental quantum mechanical laws, we would therefore need a formulation of the dynamics that does not appeal to a vague notion like "measurement," but instead only references fundamental physical properties to determine how systems are going to behave. Such a formulation would *explain why* the piecewise algorithm is as predictively successful as it is.[3]

The second concern is that textbook quantum mechanics doesn't provide us with a quantum ontology—i.e. it doesn't tell us what the world is like if quantum mechanics is true of it. For example, the central predictive algorithm makes use of something called a "wavefunction," but we are given no explanation of what this thing is. Is it a piece of fundamental ontology, a representation of our knowledge of the system, a law of nature, or something else entirely? All of these proposals have been defended in the literature[4], but there is no consensus—and even if there were, we wouldn't have textbook quantum mechanics to thank for it, since it makes no effort to tell us anything about the nature of the wavefunction beyond describing its role in the dynamics.

These two issues—the measurement problem and the ontology problem—combine to suggest that textbook quantum mechanics is radically incomplete as an account of physical reality. If so, it fails to provide us with a possible fundamental physical theory for *any* world, let alone the actual one. It's because of these problems that Carroll (2019), echoing Feynman (1964), laments that physicists "can't really be said to understand quantum mechanics."

Here I want to evaluate what can be done to address these two problems given a Humean metaphysics of laws of nature. In particular, I'll argue here that Humeans about laws should regard textbook quantum mechanics as a perfectly complete physical theory, and that neither the measurement problem nor the ontology problem are reasons for dissatisfaction. Part of my case will rely on a recent paper (Dorst 2021) that argues that the measurement problem dissolves

[3]To be sure, there are a variety of different formulations of the measurement problem in the literature. For a sampling, see, e.g., Albert (1992: 73-79), Hughes (1989: 278), Ismael (2015), Lewis (2016: 49-50), and Maudlin (1995: 7). Dorst (2021) argues that the above concern about the character of the fundamental laws is what motivates all of them.

[4]See Albert and Ney (2013) for an excellent selection of essays defending some of the more realist options.

on a Humean understanding of laws. Taking most of that discussion onboard here, my primary focus will be on the second worry, namely the theory's lack of a fundamental ontology. I will argue that, from a sufficiently refined Humean perspective, that worry is misplaced.

None of this, of course, is meant to suggest that textbook quantum mechanics is *true*, nor that it faces no conceptual difficulties whatsoever. But it *does* suggest that the theory is not radically incomplete, and that, contra Carroll, physicists who have adopted it may understand it perfectly well. It also suggests an argument for Humeanism: given the physics community's robust and widespread acceptance of textbook quantum mechanics over the last 100 years, Humeanism's ability to take that theory at face value is a strong point in its favor.

To make my case, I'll be drawing on two recent developments in the Humean tradition: the Package Deal Account (PDA)[5] and the Best Predictive System Account (BPSA).[6] In doing so there is a risk that I am winnowing my audience quite a bit: "Textbook QM is complete, but only for Humeans, and only for Humeans who accept both the PDA and the BPSA." While I won't have much to say in favor of Humeanism more generally, I will argue that Humeanism is best developed in a way that merges both the PDA and the BPSA (though the details of this merger are complex—see §4). So I hope that my argument is compelling to anyone with Humean sympathies.

Here is how this paper proceeds. Sections 2 and 3 explain the PDA and the BPSA, respectively. Section 3 also reviews the argument from Dorst (2021) that the BPSA dissolves the measurement problem. This removes one of the two reasons for regarding textbook QM as incomplete, but leaves the ontology problem untouched. The rest of this paper is concerned with dissolving that problem. Section 4 argues that the PDA and the BPSA are made for each other: the most promising way of developing Humeanism requires resources drawn from both views. Section 5 then argues that the resulting Humean position implies that textbook QM is a complete physical theory that is worth taking seriously, and in particular that the lack of a clear fundamental ontology is a misplaced objection. Section 6 explores the upshots of these arguments for Humeanism more generally.

On its surface, then, this is a paper about quantum mechanics. But its *real* concern is Humeanism. My aim is to use textbook quantum mechanics as a case study to investigate where Humeanism, properly developed, is headed.

2 The Package Deal Account

Both the Package Deal Account and the Best Predictive System Account take David Lewis's Best System Account (BSA) as their starting points. This section briefly reviews the BSA before explaining and motivating the PDA.

[5]Developed in Loewer (2007, 2020, ms).

[6]Developed in Dorst (2019).

2.1 The Best System Account

There are two primary components to Lewis's BSA (1973, 1986, 1994). First is the "Humean mosaic," the distribution of perfectly natural properties instantiated at spacetime points. Lewis placed a number of restrictions on the nature of perfectly natural properties; for example, they are supposed to be local, intrinsic, and non-modal, so that their instantiation at any particular spacetime point has no metaphysical implications for the instantiation of other perfectly natural properties at other spacetime points. Moreover, they are supposed to be the fundamental properties discovered by finalized physics.

Fundamentally speaking, the Humean mosaic is all there is to the world according to the BSA. So the laws of nature are not fundamental; rather, they are a result of systematizing the mosaic in a way that best balances the competing virtues of simplicity and strength (i.e. informativeness) and refers only to perfectly natural properties. The idea, roughly, is that the laws constitute the most *efficient summary* of everything that happens in the mosaic. If God were telling you as much about the world as possible in a short amount of time, she would give you the members of the best system.[7]

These systematizing standards are the second component of the BSA. They jointly comprise a "nomic formula" that takes the mosaic as input and returns the laws as output. Lewis (1983: 41) suggests that the standards inherent in the nomic formula come from actual scientific practice: they are the standards that scientists appeal to in their attempts to figure out the laws. The BSA thus aims to take scientists' epistemic standards for lawhood and elevate them to the status of constitutive standards. (I say it *aims* to do this because, as we will see in §3, the BSA plausibly misidentifies what scientists' epistemic standards really are.) One reason this is an attractive maneuver is that if the standards are correctly identified, it obviates the need to provide an independent justification for the reliability of those standards as guides to the laws.

The BSA accommodates probabilistic laws by allowing the best system to posit chances. The idea is that chances may serve the aim of producing the most efficient summary of the mosaic. For simplicity, consider a mosaic that consists of just a series of coin flips. One candidate system would just report every outcome in order; another would simply say "there are coin flips" and not say anything about the outcomes. The first is extremely informative but complicated; the latter is quite simple but uninformative. A better balance may be achieved by a system that describes the coin flips as chancy, e.g. perhaps it says that each coin flip has an equal chance of coming up heads or tails. This would lead us to expect a roughly equal frequency of outcomes. If indeed that is what happens, then the systematization that posits chance may be the best, thereby making the corresponding chance claims true.[8] Chance, on the BSA, is therefore a kind of *manufactured structure*, introduced by the best system in

[7] Albert (2015: 23), Beebee (2000: 574), and Lange (2009: 101-102) present the BSA using this kind of theological analogy.

[8] This sketch of the BSA's treatment of chance would need to be fleshed out in a number of ways. See Hall (2020, msb) for some instructive suggestions and worries.

order to better satisfy the standards in the nomic formula.

The BSA captures the basic Humean idea that while there are pervasive regularities in the particular matters of fact, we should not try to account for these regularities by positing unseen (and unsee*able*) whatnots—The Laws—standing *behind* those regularities and "governing" or "producing" them. Such posits will only push our explanatory demands one step backward, since any question about why the particular matters of fact exhibit such-and-such pervasive regularities can be re-posed at the level of the anti-Humean whatnots that are supposed to be producing them. Instead the Humean acknowledges that explanations come to an end somewhere (with a furtive glance toward Wittgenstein, perhaps), and takes the pervasive regularities to be criterial for, rather than merely symptomatic of, the existence of the laws.[9]

There are a wide variety of objections that have been raised against the BSA. The two that will be the most important for our purposes are the Mismatch Objection and the Pragmatic Objection. I'll focus on the former here and the latter in §3.

The roots of the Mismatch Objection trace back to van Fraassen (1989), though its name is due to Demarest (2017), who puts it in its most trenchant form. The worry is that given Lewis's restrictions on the nature of perfectly natural properties, it is conceivable that there turns out to be a mismatch between the properties posited by our final, complete theory of physics and those required by Lewis's metaphysics. We might even imagine physicists discovering such a theory and excitedly announcing, "We've got something really nice here. We're calling it the Theory of Everything." And then a Lewisian interjects, "Ahem, sorry, but actually this is no good. The properties posited by your theory are not perfectly natural."

Loewer (2020: 13) regards this mismatch possibility as a significant worry for the BSA:

> [I]t seems presumptuous for a metaphysician to say to a physicist who believes she has found a theory that optimally satisfies all the scientific criteria but not the metaphysical one[s] that she may not have discovered the laws since the theory is not formulated in the language of perfectly natural properties. If one thinks, as I do, that it is physics not metaphysics that determines where nature's joints are this response is not appealing.

Loewer's thought, I take it, is that rejecting a widely-accepted physical theory because it doesn't accord with one's metaphysical commitments is objectionably hubristic. Philosophy should not be trying to dictate to physics what the latter's theories must be like. One of the main motivations for Loewer's Package Deal Account is to remove the potential for this mismatch between the deliverances of physics and those of metaphysics.

[9] I don't mean to imply that Humean laws cannot explain the regularities. But the sense in which Humean laws would explain them is quite different from the sense in which anti-Humean laws would. See, e.g., Loewer (2012), Bhogal (2020), and Duguid (2021) for discussions about how to understand nomothetic explanation from a Humean perspective.

2.2 The Package Deal Account and Its Virtues

The core idea of the PDA is to treat *all* of the physically fundamental properties, as well as the world's spacetime structure, in the way the BSA treats chance: as structures manufactured by the best system. The aim is to design a metaphysics of laws, fundamental ontology, and spacetime structure in a way that mirrors their epistemology. In actual scientific practice, physicists' posits in each of these categories can inform, and be informed by, their posits in the others. As Cohen and Callender put it,

> [S]cientists actually devise laws based on their choice of kinds and choose their kinds based on the laws. Gell-Man[n] was not simply handed fractional charge and left to make the best system he could with it; rather he postulated fractional charge in part because he saw that he could make a very simple and strong system with it if he did. (2009: 13)

The same, of course, is true with respect to the spacetime structure. For example, the dynamical implications of the laws can be greatly affected by the nature of the spacetime in which they are operating; posits about the laws and spacetime structure are thus mutually interdependent.

In actual physical practice, then, the laws, fundamental ontology, and spacetime structure are all on a par, epistemically speaking: we do not have privileged access to one of them over the others. The PDA attempts to mirror this epistemic symmetry in its metaphysics by saying that the laws, fundamental ontology, and spacetime structure are all determined jointly, as a package deal, by the best system.

Moreover, the PDA maintains Lewis's idea that the nomic formula should be comprised of the epistemic standards used in actual physical practice. This is how it handles the Mismatch Objection. If the truths about the laws, fundamental ontology, and spacetime structure are determined by the same standards used in physical practice, there is no possibility of a divergence between the ontology of a final theory of physics and that of our metaphysics.

A natural question at this point is that if the fundamental ontology is treated as a *product* of the best system, what is the best system supposed to be systematizing? What, in other words, are the laws, fundamental ontology, and spacetime structure posited in response to?[10] This is a significant interpretive question, and I will suggest an answer that I think is best dialectically and—*perhaps*—exegetically.[11] Loewer hints at this answer in a paper from almost thirty years ago:

> A proposal that I think captures what many have on their minds when they speak of fundamental physical properties is that they are the properties expressed by simple predicates of the true comprehensive fundamental physical theory. The true comprehensive

[10] Cf. Demarest (2019: 393).

[11] For present purposes, I am less concerned with whether Loewer himself understands the PDA in this way, though he certainly says things that are indicative of it.

> fundamental physical theory is the minimal theory that accounts for the changes of locations and motions of macroscopic spatio-temporal entities and also for changes in properties that account for locations and motions and so on. (1996: 110)

This passage is remarkable for its nonchalance in the process of making a fantastically bold proposal. The true comprehensive fundamental physical theory, on this view, is the one that best accounts for the behaviors of macroscopic entities, where "best accounts" is understood to mean that it accounts not only for the behaviors of the macroscopic entities directly, but also for the behaviors of the things posited to account for the behaviors of the macroscopic entities, etc. Loewer continues to espouse this idea in more recent work, sometimes describing the objects of systematization as the *macroscopic phenomena* (Loewer, 2020), which collectively comprise the *macroscopic mosaic* (Loewer, ms). On this interpretation of the PDA, the laws, fundamental ontology, and spacetime structure are determined as a package deal by the best systematization of the macroscopic phenomena.

This proposal raises a number of difficult questions right off the bat. Among them: Are macroscopic phenomena sufficiently nonmodal to count as Humean? Can the macroscopic phenomena be characterized without presupposing nomic concepts? Where are the borders of the macroscopic? And most importantly, *what's the motivation for this?* There are far too many questions for me to attempt to answer them all here[12], but I will try to answer the last of them, and hopefully that will help to shed some light on the others.

Dialectically speaking, there are three main reasons I think that the conception of the mosaic as comprised of the macroscopic phenomena is appropriate. First, moving the physically fundamental properties out of the target of systematization and into the product of the systematization helps to explain why we are interested in the physically fundamental ontology in the first place. To see this, contrast Loewer's approach with Lewis's. If the laws are given by the best systematization of the distribution of perfectly natural properties (as on the BSA), then as Dasgupta (2018) makes clear, we need some account of why we should care about the perfectly natural properties. More specifically, in the present context we should ask why we ought to care about expressing the laws in terms of them. If, following Lewis, we think of perfect naturalness as a metaphysical primitive that constrains—by *fiat*—how the laws may be expressed, then no answer is forthcoming. By contrast, on the PDA the role of the physically fundamental properties is not to constrain the best system, but rather to augment it. The value of identifying the physically fundamental properties, and expressing the laws in terms of them, is thus directly tied to the value of discovering the best system in the first place. So long as *that* is valuable, we will have an account of the value of the physically fundamental properties.[13]

[12] Some of Loewer's own answers can be found in Loewer (2007, 2020, ms). Relevant and helpful discussions can also be found in Demarest (2019) and Eddon and Meacham (2015).

[13] This is not meant to imply that there isn't a significant question about the value of systematization. We'll return to this issue in §3.

Of course, discovering the best system *won't* be valuable if we have no reason to care about the stuff getting systematized in the first place. And this is why it makes the most sense for that stuff to be the macroscopic phenomena: we *live amongst* those phenomena, and our interests depend on our being able to navigate them effectively. Dasgupta himself makes a similar suggestion, raising the possibility that

> the privileged properties, the ones on which one's metaphysics is built, are not the properties of fundamental physics but rather those revealed in perception. Perception reveals the maximally 'elite' properties; other properties like those of physics are derivative upon these. I do not know how popular this kind of view would be among contemporary realists. But it is the only realist view I can think of that might avoid the problem of missing value. (2018: 319)

While Dasgupta ties his considerations more directly to perception, the general point is surely friendly to this way of understanding the PDA.

Second, since the PDA's standards for fixing the truths about the laws, fundamental ontology, and spacetime structure are supposed to be the epistemic standards employed in physical practice, it is only natural that the PDA applies them in the same way that physicists do. As Loewer (1996, 2020, ms) points out, when practicing physicists apply these standards, they do so to systematize data they've gathered about the behavior of macroscopic phenomena. *These* are the kinds of data that our physical theorizing is ultimately answerable to.

If physicists' epistemic standards are designed to be applied to the macroscopic phenomena, then it would be quite odd for a philosophical view of laws to recommend adopting those standards but applying them to something else. This is essentially what the original BSA does: it takes physicists' epistemic standards for lawhood and recommends applying them to the distribution of perfectly natural properties. But those properties themselves are supposed to be discovered by physics. If so, the view is essentially saying that physicists' epistemic standards get applied *twice*: first to discover the perfectly natural properties, and second to discover the laws as the best systematization of the perfectly natural properties. This assumes that the standards can be applied piecemeal to different kinds of theoretical posits, but as Cohen and Callender's example of fractional charge illustrates, posits in different categories depend on one another. The BSA thus recommends applying physicists' epistemic standards in a way they are not designed to be applied. The same would be true of any other account that adopted those standards but tried to apply them to kinds of things that physicists themselves do not.

The third reason to think that the PDA's choice of mosaic is correct is that it makes the same sort of move with respect to the fundamental physical ontology that the Humean is already committed to with respect to the laws. Faced with pervasive regularities in the particular matters of fact, the Humean takes these to be constitutive of the laws rather than merely indicative of them. Likewise, faced with pervasive regularities in the behaviors of macroscopic phenomena, the PDA (on this reading) takes these to be constitutive of the truths about their microscopic constituents rather than merely indicative of the existence of such constituents. In both cases, there is a pretheoretic temptation to posit something operating "behind the scenes" that produces the behavior of the scenery. But in both cases, all of the questions we are tempted to raise about the behavior of the scenery—and to explain by reference to things that are offstage—could be raised again with respect to the latter. Just as we may ask why the macroscopic phenomena behave the way they do, we may likewise ask why the microscopic constituents of the macroscopic phenomena behave the way that they do. Of course, this doesn't show that a theory of laws and fundamental physical ontology *has* to apply the same constitutivization move in both cases. Coherent theories, such as the BSA, have been constructed that apply it to only the laws, for example. But such views end up drawing a very tortuous line by treating these similar cases so differently. In treating the two cases alike, the PDA exhibits a more uniform commitment to some of Humeanism's most basic motivations. On the PDA, fundamentally speaking, there is nothing behind the scenes; *all* the world's a stage, not just part of it.

The resulting view is one according to which the notions of metaphysical and physical fundamentality are going to come significantly apart. The metaphysically fundamental ontology consists of the macroscopic phenomena that comprise the mosaic prior to any systematization, whereas the physically fundamental ontology is manufactured by the best system.[14] We thus end up with a version of what Bernstein (2021) calls "middleism," according to which the metaphysically fundamental ontology occupies a middle level (in terms of size), and grounds the facts about both lower and higher levels.

Bernstein defends middleism from a number of objections, arguing that it is no worse off than either "bottomism" or "topism" in terms of a priori desiderata for a view about the structure of reality. However, she admits that it may be difficult to use the notion of size to articulate precisely where the fundamentalia are to be found according to middleism. (If middleism is correct, are forks fundamental? Or is it rather the individual tines on the forks?[15]) She responds that there is no requirement that facts about what's fundamental have to be *delineated* according to size; rather it's just that we often use size as a convenient, albeit imprecise, way to pick out what is fundamental.

The fan of the PDA can accept this answer, but also add that the boundaries of what is metaphysically fundamental will be determined by physical practice. What, ultimately, are the kinds of facts that physical theorizing has to answer to? Whatever the answer—and there must *be* an answer, if physics is to proceed sensibly—the PDA will take them to be metaphysically fundamental.

[14]This divergence in notions of physical and metaphysical fundamentality might bring with it a divergence in physical and metaphysical *possibility*, for the possibilities we countenance by focusing on the physically fundamental ontology might be very different than the possibilities we countenance by focusing on the metaphysically fundamental ontology. Bhogal (2020) has argued persuasively that the Humean ought to embrace this sort of divergence in physical and metaphysical modalities.

[15]This question is courtesy of David Builes.

3 The Best Predictive System Account

In §2.1 I mentioned two objections to Lewis's BSA: the Mismatch Objection and the Pragmatic Objection. We saw how the former helps to motivate the PDA. Let's now consider how the latter motivates the BPSA.

Humeanism attempts to reduce the laws to some sort of patterns in the totality of the particular matters of fact. Given this basic outlook, Hall (msa) has persuasively argued that Humeans owe us an account of the laws' scientific significance. Why are the laws, to use Hall's words, "distinctively appropriate targets of scientific inquiry"? Patterns in the particular matters of fact are a dime a dozen, so something more needs to be said to explain the laws' prominent place in our scientific investigations.

The natural answer from the Humean perspective is to appeal to the practical utility of the laws. If the laws are patterns that are particularly useful to creatures in our epistemic situation, then it would be quite reasonable for science to place a premium on their discovery despite their metaphysical insignificance. But this raises a worry about Lewis's BSA, namely that its laws, qua efficient summaries, are not particularly useful to creatures like us. Dorst (2019) makes this point by analogizing our position in the mosaic to that of a person trying to navigate an enormous maze. From such a perspective, it would not be particularly helpful to be told facts that efficiently summarize the maze (e.g. its total acreage, or the average amount of time people spend lost in it). Likewise, it would not be particularly helpful for us to be told an efficient summary of everything that happens in the total history of the universe. If this analogy is apt, the BSA fails to explain why laws are distinctively appropriate targets of scientific inquiry.

A number of recent developments of the BSA have focused on this shortcoming and attempted to reconceive the goal of systematization so that it is more useful to creatures like us. In doing so, they have converged on the idea that the best system is the one that is *optimally predictively useful* to creatures in our epistemic situation. This change in the goal of systematization engenders a corresponding change in the nomic formula. For example, the Best Predictive System Account (Dorst 2019)[16] replaces the standards of simplicity and strength with a set of "predictive desiderata" that are meant to produce systems that are predictively useful. We needn't go into the details here, but the general idea is that the best predictive system will have an amplifying function: it will take input information about any given physical system of interest and output a great deal more information about the behavior of that system over time. Moreover, the sorts of information that the best predictive system requires in order to generate its outputs ought to be those to which we tend to have access. The standards in the BPSA's nomic formula are designed to produce a system like this.

The BPSA was developed to provide a straightforward answer to the Prag-

[16]For simplicity, I am focusing here on the BPSA, but similar views can be found in Callender (2017), Hicks (2018), Jaag and Loew (2020), and Wilhelm (2022). Some of the following discussion might be sensitive to the differences between these views.

matic Objection, but it also bears on our present concerns with textbook QM, as it appears to dissolve the measurement problem. The basic thought (developed in Dorst 2021) is that the measurement problem relies on an anti-Humean conception of laws, since the idea that the piecewise dynamics from §1 cannot be correct hinges on the claim that the laws of nature cannot be *paying attention* to whether or not systems are being measured, and *directing* their behavior accordingly. But on a Humean view, laws neither "pay attention" to what systems are doing nor "direct" the behavior of those systems. They are merely descriptions of the patterns of behavior of those systems—descriptions satisfying certain standards. If the BPSA is correct, they are descriptions that are maximally predictively useful to us.

Granted, textbook quantum mechanics gives us a somewhat vague dynamical recipe, since the boundaries of exactly when a measurement occurs are imprecise. But this vagueness would only be a problem, according to the BPSA, if it adversely affected our predictive capabilities. And textbook quantum mechanics is, for all practical purposes, predictively equivalent to any of the more precise interpretations, like GRW or Many Worlds or Bohmian Mechanics. (Indeed, this is what makes it so hard to distinguish between those theories on empirical grounds.) So the vagueness of "measurement" is an objection that does not gain traction from the perspective of the BPSA.

If this is right, it undercuts one of the two reasons for thinking that textbook quantum mechanics is incomplete as a physical theory. To address the second reason—the lack of a quantum ontology—we need to merge the PDA and the BPSA (though we will see that it is almost more of a takeover than a merger). That is the project of the next section.

4 A Meta Package Deal

The PDA and the BPSA concern different aspects of the Humean picture. Broadly speaking, the PDA focuses on the nature of the mosaic and manufactured structures, and the BPSA focuses on the nomic formula. Given their different foci, it remains an open possibility that they could be conjoined.

One quick way to motivate this conjunction is to note that adopting either of these views without the other leaves unaddressed some of the problems that each is designed to handle. For example, the PDA without the BPSA leaves it mysterious why we should care about systematizing the macroscopic phenomena according to the standards of simplicity and strength—why care about an efficient summary of the totality of macroscopic phenomena?[17] Likewise, unless the BPSA adopts the PDA's conception of the physically fundamental

[17] Loewer (2020) suggests that further standards, drawn from physical practice, should be added into the nomic formula, and in so doing he mentions a number of the standards that Dorst appeals to in articulating the BPSA. If we construe this amendment to the nomic formula to be an official component of the PDA, this brings it quite close to the combined position I'm advocating here. What the BPSA explicitly adds, in that case, in an explanation of the *point* of systematizing according to those standards—namely to design the best predictive system—thereby explicating what is meant by a system achieving the "best balance" of them.

ontology—namely, as a set of manufactured structures that serve the aim of systematizating the macroscopic phenomena—it will be unclear why we ought to care about it and why we should want the best predictive system to reference it.

There's also a deeper sense in which the BPSA can be seen to motivate the PDA. At a very basic level, both views take there to be a mosaic that gets systematized, and so both views are going to face the question of what the *point* is of systematizing in the first place. The PDA doesn't directly provide an answer to this question. Rather, it takes for granted that systematization is occurring and focuses on (i) what is getting systematized, and (ii) what sorts of structures get introduced as part of the systematization. The BPSA, by contrast, directly addresses this question: the point of systematization is to produce a system that is maximally predictively useful to creatures in our epistemic situation.

Suppose, then, that the best predictive system were *not* allowed to introduce the kinds of manufactured structures that the PDA allows it to, but instead had to systematize the mosaic by appealing only to the kinds of structures that are already part of the mosaic. Call the best system that can be achieved within these constraints $\mathcal{S}_1$. As far as I can see, there is no way to rule out the possibility that a different system that *did* introduce new structures would be better for predictive purposes. Call such a system $\mathcal{S}_2$. In that case, insisting that we stick with $\mathcal{S}_1$ when $\mathcal{S}_2$ is available appears entirely unmotivated from the most basic commitments of the view, according to which the point of systematizing in the first place is to secure the optimal predictive system. If $\mathcal{S}_2$ does better than $\mathcal{S}_1$ in this regard, so be it.

I'll note in passing that there is a significant question about what sorts of mosaics motivate the introduction of what sorts of structures in the service of predictive utility. The tenability of merging the PDA and BPSA depends on the answer, because in order for us to evaluate that view, we need to know whether the sorts of structures that it recommends are the sorts of structures posited in physical practice. This is clearly a large project, however, and there isn't enough space here to adequately carry it out—a not altogether unhappy fact, since I'm not entirely sure how to do it (though I'll speculate a bit below).

Admitting, then, that there are significant outstanding questions about this view's tenability, what will it ultimately look like? For starters, it will have a grotesque acronym: the "BPSPDA." More substantively, the view will be one on which the laws, fundamental physical ontology, and spacetime structure are all posited as a package deal by the best predictive systematization of the totality of macroscopic phenomena. This is a radical revision of the orthodox BSA. Let's look at some of its implications.

The BPSPDA supports the following thesis about the structure of our fundamental physical theories:

> **First-Order Opportunism:** There are no a priori constraints on the nomological, ontological, or spacetime structures posited by the best predictive system.

The best predictive system may posit whatever kinds of structures it wants within each of these categories in order to secure optimal predictive utility. The structures that turn out to be optimal in this regard are ultimately going to be a function of what the macroscopic mosaic is like. This means that there is no a priori reason to require that the fundamental physical ontology be comprised of, say, perfectly natural properties instantiated at spacetime points (contra Lewis), or particles (contra Esfeld, 2020), or whatever else; it could just as well be fields, strings, foam, etc. at the level of fundamental physics. Likewise, there is no a priori requirement that the spacetime structure must consist of, say, three-dimensional Euclidean spaces ordered along a single temporal dimension. And lastly, the laws can have whatever form they please so long as the resulting system is predictively optimal.

I take it to be fairly uncontroversial that both contemporary physics and contemporary philosophy of physics would endorse First-Order Opportunism. Both physicists and philosophers take seriously theories that posit *all sorts* of different structures in each of these categories. Examples could be produced ad nauseam, though it would be pedantic to rehearse them here.

But the BPSPDA also supports a more controversial thesis. To see this, recall that on Lewis's BSA, not every system is required to posit chances. There are chances only if the best system says there are—that is, only if chances earn their keep by making the best system best. The existence of manufactured structures like chance is therefore highly contingent. Now since the BPSPDA is much more liberal about what sorts of structures may be introduced, it must likewise be more liberal about what sorts of structures *may not exist* if their introduction turns out not to contribute to the aim of systematization. It therefore supports the following thesis about the form of our fundamental physical theories:

> **Second-Order Opportunism:** There are no a priori constraints on the ontological categories employed by the best predictive system.

By that, I mean that it should not be an a priori requirement that the best predictive system introduce something in precisely each of the three ontological categories we have been considering, namely laws, fundamental physical ontology, and spacetime structure. Whether it needs something in each of these categories, or even in some other hypothetical categories, is an empirical matter that depends on what the macroscopic mosaic is like.[18]

We can make this point by again considering two candidate systems, $\mathcal{S}_3$ and $\mathcal{S}_4$. Suppose $\mathcal{S}_3$ introduces laws, fundamental ontology, and spacetime structure whereas $\mathcal{S}_4$ only introduces laws and spacetime structure (but no fundamental ontology), and yet $\mathcal{S}_4$ ends up being more useful for predictive purposes. It would then seem entirely unmotivated to maintain that we *must* opt for $\mathcal{S}_3$ because it introduces structures in each of those categories whereas $\mathcal{S}_4$ does

[18]Maudlin (2013), for example, has suggested that we take seriously the possibility that the quantum wavefunction is a sui generis entity that is not straightforwardly classified in one of the aforementioned categories. Likewise, many contemprary cosmologists regard spacetime as emergent rather than fundamental. I take these suggestions to be friendly to Second-Order Opportunism.

not. To be sure, you could stubbornly insist that the best system must posit structures in each category. But this would be akin to Lewis's insistence that the laws be formulated solely with respect to perfectly natural properties: it adds in a constraint by hand that is idle at best—and harmful at worst—by the lights of the basic motivations for the view.[19]

Indeed, we should expect that the more variegated and heterogeneous the macroscopic phenomena are, the less predictive benefit there will be to positing a fundamental ontology. If the macroscopic phenomena are all easily interpretable in terms of a fundamental particle ontology, then particles it is. But suppose some phenomena suggest particles, others suggest waves, and still others suggest nothing clear whatsoever. In that case the best predictive system of the totality of these phenomena is either going to have to settle on an ontology that can "shape" itself in many different ways, or forgo an ontology altogether.

What we're seeing here is that even though this was advertised as a merger of the BPSA and the PDA, it is really the BPSA that's in the driver's seat. This is because the BPSA provides a story about the *point* of systematizing the mosaic in the first place, whereas the PDA is concerned with the kinds of structures that can be posited to aid the systematization. Everything the PDA does, then, is in service of the goal articulated by the BPSA. If that goal is best accomplished by a system that omits, say, a fundamental physical ontology—well, then so be it.

Radical as this view is, it follows very naturally from the Humean's basic commitments. One way of trying to resist it would be to maintain that predictive utility is indeed *part* of the goal of systematization, but that there is another goal as well, namely providing an accurate representation of the fundamental physical ontology. In that case, a system that omits any such ontology is ipso facto inadequate, and it would make good sense to demand that the BPSPDA posit such structures.

This approach strikes me as fundamentally confused. According to the BPSPDA, there simply *are no* facts about the fundamental physical ontology independent of the best predictive system. There is nothing antecedently *there* for the system to represent, and any structures it posits are introduced not fundamentally as a representation, but as a predictive aid. Even if you were convinced that, independently of any considerations about predictive utility, there are facts about the fundamental ontology that we could try to represent—well, why then should our systematization procedure be forced to accommodate *both* the aim of predictive utility and the aim of representing the fundamental physical ontology? Granted, insofar as they align perfectly, there is no worry; both projects may be pursued under the same umbrella. But as soon as these

[19] Of course, Lewis's constraint wasn't altogether unmotivated. He introduced it because he was worried that without it, his view would automatically deem $\forall x Fx$ the best system (where "F" is a predicate which applies to all and only the entities in the world in question). The proper way to handle this worry, however, is not to stipulate an ad hoc constraint on the best system's language, but rather to reconceive the goal of systematization so that $\forall x Fx$ has no hope of being the best system in the first place. If we maintain that the best system is the one that is predictively optimal for creatures like us, then, I submit, we have secured that result without the ad hockery.

aims start to diverge, to force the best system to accommodate both is just to ensure suboptimal results for both. In that case, our aims would be better served by simply having two distinct projects. (And come to think of it, *shouldn't* we be pursuing the project of finding the best predictive system for creatures like us? Presumably, *hopefully*, we are already doing that.)

5 Taking Textbook QM at Face Value

We've just seen that the BPSPDA allows that the best predictive system need not posit something in each of the three categories that the PDA suggests. This bears directly on the tenability of textbook quantum mechanics as a complete physical theory, since the last remaining obstacle to taking it seriously is that it does not posit a clear quantum ontology. Now in fact there are really two distinct (but related) questions here:

1. What kind of thing is the wavefunction?
2. What is the fundamental quantum ontology?

Textbook quantum mechanics, as I am construing it here, answers neither of these questions. What we need to consider is whether it *has* to answer them in order to be taken seriously as a physical theory.

Given the BPSPDA, in one sense the answer is immediate. If the BPSPDA implies Second-Order Opportunism, then it places no a priori constraints on the form of a legitimate physical theory. So textbook quantum mechanics is not eliminated as a potential best system simply because it doesn't provide an answer to (1) or (2). Still, the substantive question here isn't whether textbook QM technically qualifies as a physical theory, but whether it's one worth taking seriously as a theory of the actual world.

The received wisdom among philosophers of physics, apparently, is no.[20] For example, supporters of the primitive ontology approach (as articulated in, e.g., Allori et al. 2008) will maintain that we need an account of the theory's ontology before we can know what predictions it makes about what experiences we will have. Here, for example, is Maudlin discussing what he views as the shortcomings of textbook QM in the introduction to his (2019) book on quantum theory:

> A physical theory should clearly and forthrightly address two fundamental questions: what there is, and what it does. The answer to the first question is provided by the ontology of the theory, and the answer to the second by its dynamics. The ontology should have a sharp mathematical description, and the dynamics should be implemented by precise equations describing how the ontology will, or might, evolve. All three of the theories we will examine [GRW, Bohmian Mechanics, and Many Worlds] meet these demands. [Textbook quantum mechanics], in contrast, does not. There is little

[20]Of course there are exceptions—Healey (2017) being one of the most notable.

> agreement about just what this approach to quantum theory postulates to actually exist or how the dynamics can be unambiguously formulated. (2019: xi)

Maudlin here articulates an extremely compelling and intuitive picture of what physics is up to: physical theories provide us with both a precise ontology and a precise dynamics describing the ontology's behavior. But what is the *argument* for this picture? It cannot be that the history of physics unambiguously supports it, for textbook QM, with its vague dynamics and lack of an ontology, does not. If instead the argument is that without these elements, we would be unable to explain why the macroscopic world we inhabit behaves the way that it does, then the Humean should be suspicious, for their general strategy for explaining regularities is not to posit things "behind" them, but rather to systematize them in a way that makes their behavior more predictable.

Contrary to the received wisdom in philosophy of physics, the Humean view I have been articulating here recommends the heretical idea that we take textbook QM seriously, notwithstanding the lack of a clear ontological picture. That's because, as we saw in §3, it is for all practical purposes predictively equivalent to more precise interpretations like GRWm, GRWf, Many Worlds, and Bohmian Mechanics. After all, it's notoriously difficult to distinguish between these theories on empirical grounds. This implies that the precise ontologies of these alternative theories are not pulling their predictive weight—else they would ramify into different empirical predictions.[21] But then the BPSPDA will find no reason to recommend them over the sparser, vaguer systematization provided by textbook QM. From the point of view of the BPSPDA, each of these alternative theories just posits superfluous ideological superstructure.[22]

Nor is it a legitimate objection that textbook QM posits a wavefunction without telling us what kind of thing it is. Just as Hall (2020, msb) argues that Humean chances need not have perfectly precise real-numbered values, so too we can allow that entities like the wavefunction need not occupy perfectly precise ontological categories. The question, again, is whether classifying the wavefunction as ontic, nomic, epistemic, or something else entirely, is helpful for predictive purposes. Insofar as practicing physicists are able to use the piecewise predictive recipe without engaging such questions, the answer, again, will be negative.

"But," you might protest, "the resulting view doesn't tell us what the world is like if textbook QM is true of it. How can we be satisfied with such a theory?" Tempting as this worry is, ultimately I think it is misguided. The BPSPDA *does* tell us what the world is like if textbook QM is true of it: the world is composed of macroscopic entities that behave in a way best systematized by textbook QM. By any plausible metric of charitability, this surely counts as one of the most

[21] There are subtleties here, of course. For example, certain values of the GRW localization accuracy and frequency parameters would have experimentally detectable consequences. But there is currently a range of possible values of those parameters that we *cannot* distinguish experimentally. See Feldmann and Tumulka (2012) for further discussion.

[22] This is Heisenberg's (1958: 133) description the ontology of Bohmian Mechanics as compared to textbook QM.

charitable interpretations of textbook QM available. Anyone who tells you that it couldn't possibly be correct is in the grip of a picture—a compelling one, no doubt—that is suggested by some, *but not all*, of the prominent theories from the history of physics. If my arguments here are on the right track, it's a picture that the Humean's basic commitments give them reason to resist.

To be sure, many philosophers of physics will want to turn my arguments here around: if the BPSPDA suggests that we should take textbook QM seriously as a genuine physical theory, so much the worse for the BPSPDA. By and large, however, practicing physicists *do* appear to take textbook QM seriously. Here, again, is Sean Carroll describing the situation:

> Few modern physics departments have researchers working to understand the foundations of quantum theory. On the contrary, students who demonstrate an interest in the topic are gently but firmly—maybe not so gently—steered away, sometimes with an admonishment to 'Shut up and calculate!' Professors who become interested might see their grant money drying up, as their colleagues bemoan that they have lost interest in serious work. (2019)

It is tempting to interpret physicists' satisfaction with textbook QM as an indication of the sorry state of contemporary research in fundamental physics. But this interpretation is not mandatory. One might instead view this as a case where the physics community is fairly univocally saying, "We've got something really nice here in textbook quantum mechanics," and philosophers of physics are saying, "Ahem, sorry, no you don't. Your theory doesn't conform to our metaphysical requirements." If we thought that this kind of move was objectionable in the case of the Mismatch Objection, we should at least consider the possibility that our dismissive attitudes toward textbook QM are analogous. In that case, Humeanism's ability to take textbook QM at face value would not be a serious mark against it, but a significant point in its favor.

6 Conclusion

But what exactly *is* Humeanism at this point? While there is certainly a resemblance between the BPSPDA and Lewis's original BSA, you have to squint to see it. To some extent, this is true of contemporary Humeanism more generally. In the years since Lewis first suggested that chance could be a manufactured structure, there has been a proliferation of similar moves by other Humeans. Hall (msa) has suggested both that properties like mass and charge could be thought of in this manner, and even that they needn't have perfectly precise values (2019, msb). Likewise, Miller (2014) and Bhogal and Perry (2017) suggest thinking of the wavefunction as a manufactured structure. Hicks and Schaffer (2017) argue that Humean laws can reference derivative properties, and Chen (2022) that they can reference vague ones. And Loewer, of course, has argued that *all* of the physically fundamental structures should be thought of as manufactured by the best system.

The writing, it seems, is on the wall. Humeanism's basic commitments allow it to do more or less whatever it wants in order to serve the aim of systematization. And we've seen that that aim itself needs to be construed pragmatically in order to avoid the Pragmatic Objection. On the resulting view, physical theories should be thought of not as representations of the world as it is in itself, but as tools designed in response to our practical concerns. To put it brusquely, Humeanism is starting to look like a kind of glorified instrumentalism.

I think this is more or less correct. (This paper has hidden one heresy behind another.) To be sure, it's an instruentalism that allows our theoretical claims to be *objectively true*—even those that might concern unobservable structures—though these truths are ultimately derivative upon their predictive utility to creatures like us. For a while now, Humeanism has been trying to occupy a sort of middle ground between an out-and-out realism, on the one hand, and a thoroughgoing instrumentalism, on the other. But the trend is plainly toward instrumentalism, and it is not difficult to see why. Once it becomes clear that Humeanism's reductionist stance requires a pragmatic understanding of the point of systematization, it is hard to find any principled reasons to place substantive, realist-friendly constraints on how that systematization can be pursued.[23] Admittedly, there are coherent Humean views in the space between the realist and instrumentalist extremes. You can pitch your tent there. But you're camping on a slope.

[23] Cf. Miller (2014: 582).

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