How do Laws Produce the Future?

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Abstract

The view that the laws of nature produce later states of the universe from earlier ones (prominently defended by Maudlin) faces difficult questions as to how the laws produce the future and whether that production is compatible with special relativity. This article grapples with those questions, arguing that the concerns can be overcome through a close analysis of the laws of classical mechanics and electromagnetism. The view that laws produce the future seems to require that the laws of nature take a certain form, fitting what Adlam has called "the time evolution paradigm." Making that paradigm precise, we might demand that there be temporally local dynamical laws that take properties of the present and the arbitrarily-short past as input, returning as output changes in such properties into the arbitrarily-short future. In classical mechanics, Newton's second law can be fit into this form if we follow a proposal from Easwaran and understand the acceleration that appears in the law to capture how velocity (taken to be a property of the present and the arbitrarily-short past) changes into the arbitrarily-short future. The dynamical laws of electromagnetism can be fit into this form as well, though because electromagnetism is a special relativistic theory we might require that the laws meet a higher standard: linking past light-cone to future light-cone. With some work, the laws governing the evolution of the vector and scalar potentials, as well as the evolution of charged matter, for electromagnetism in the Lorenz gauge can be put in a form that meets this higher standard.

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1 Introduction

In physics, we search for laws of nature. These laws are often described as governing the evolution of systems over time—producing, or generating, later states from earlier ones. As Maudlin (2007, pg. 182) puts it:

> "The universe, as well as all the smaller parts of it, is *made*: it is an ongoing enterprise, generated from a beginning and guided towards its future by physical law."

This dynamic production view of physical laws can be contrasted with two alternatives. The first is the Mill-Ramsey-Lewis best systems account, according to which the laws do not truly govern what happens in nature, but instead merely offer concise and informative descriptions of all the things that happen throughout the entire history of the universe (including its full past and future). Put poetically, the laws are patterns in the mosaic that is our universe. The second is a recent alternative put forward by Chen & Goldstein (2022) and Adlam (2022a) according to which the laws govern without dynamic production by placing constraints on possible histories for the universe.

There is much that could be said about the advantages and disadvantages of these three competing accounts, but here I would like to focus on two serious challenges facing the dynamic production account. First, one might wonder how the laws produce the future. Second, one might be worried as to whether that production is compatible with special relativity.

Asking how the laws produce the future verges on the unanswerable, as the dynamic production account takes the action of the laws to be primitive—not something that can be analyzed in terms of anything more fundamental. But, there is a sensible question here. Chen & Goldstein (2022, pg. 60) and Dorst (unpublished) ask about the relata of the production relation: what is being produced by what? If we take the input of the laws to be the state of the world at just one moment, we only have a static snapshot that leaves out features of the world—like velocities—that are needed to determine future evolution (Chen & Goldstein, 2022, pg. 46 & 60). Specifying the output is also tricky. The output of the laws cannot be the state of the world at the next moment because (if time is continuous) there is no next moment (Loewer, 2012, pg. 133).

For the dynamic production account to be viable, the laws of nature must take a certain form. They must include dynamical laws that specify (either deterministically or probabilistically) how the world evolves from one moment to the next. The laws must fit what Adlam (2022a,b) has called "the time evolution paradigm." Making that paradigm precise, we might demand that there be temporally local dynamical laws that take properties of the present and the arbitrarily-short past as input and return as output changes in such properties into the arbitrarily-short future. For theories like this, we have a clear picture as to how the laws produce the future. The input to the laws is not just the present moment and the output is not the state at any particular future moment.

Temporally local dynamical laws are not hard to find. In classical mechanics, Newton's second law can be fit into the above form if we follow a proposal from Easwaran (2014) and understand the acceleration that appears in the law to capture how velocity (taken to be a property of the present and the arbitrarily-short past) changes into the arbitrarily-short future. In classical electromagnetism, two of Maxwell's equations can be interpreted as temporally local dynamical laws (and the other two can be interpreted as non-dynamical laws).

Let us now turn to the second challenge. Dorst (unpublished) has criticized the dynamic production account for being in tension with the special theory of relativity. One objection is that special relativity pushes us to a block universe theory where the past and future are just as real as the present and there is no work for the laws to do in producing the future. The future is already there. This objection will be addressed briefly in section 3. Dorst's other objection is that we cannot say that dynamic production is occurring from one time slice to the next without privileging a particular way of carving the universe into simultaneity slices. Maudlin is willing to introduce such a preferred foliation because he believes that in quantum physics a preferred foliation is the most natural way to account for the experiments that illustrate Bell's theorem. I think that it is valuable to understand how relativistic dynamic production might work without a preferred foliation because there are important classical theories that do not need such a foliation and because quantum theories that include parallel universes may not need such a foliation.

In a relativistic theory like classical electromagnetism, we can go beyond merely requiring temporally local dynamical laws that connect past and future and require that the dynamical laws be spatiotemporally local, connecting the past and future light-cones at every space-time point. This higher standard can be called "the relativistic time evolution paradigm." With some work, we will see that the laws of electromagnetism can be put in this form. There are three types of laws that must be analyzed. First, there are the equations describing how charged matter produces the electromagnetic field. In the Lorenz gauge, these are the relativistic wave equations for the vector and scalar potentials. Such wave equations are standardly presented alongside causality theorems proving that what happens at a given point is fully determined by what happens on any slice of its past light-cone. The wave equations can be manipulated to make this causal structure manifest so that they describe how properties of the arbitrarily-small past light-cone change along the future light-cone. Second, there is the Lorentz force law giving the electromagnetic force on charged matter. This can be rewritten so that the force on a body at a location depends only upon properties of the arbitrarily-small past light-cone. Third, we need an equation describing how charged matter responds to forces. For a point charge, this would be the relativistic extension of Newton's second law, where the force on the charge can be set equal to the rate at which the charge's relativistic momentum (interpreted as a property of its arbitrarily-short past) changes into its arbitrarily-short future. Because the charge never moves faster than the speed of light, the law will describe how properties of the arbitrarily-small past light-cone change into the future light-cone.

The article proceeds as follows: The next section presents a precise statement of the time evolution paradigm that theories must fit for the dynamic production account of laws to be tenable. Section 3 compares the dynamic production account of laws to two alternatives: the Mill-Ramsey-Lewis best systems account and the Chen-Goldstein-Adlam laws-as-constraints account. Section 4 explores multiple ways of fitting Newton's classical theory of gravity into the time evolution paradigm, highlighting some challenges and choice points that will be relevant to our later discussion of electromagnetism. Section 5 gives a simple way to cast electromagnetism into the time evolution paradigm. Section 6 puts forward the relativistic time evolution paradigm. Section 7 presents a way of fitting electromagnetism into the relativistic time evolution paradigm, showing that it is possible and leaving open whether there might be a better way of doing so. The final section concludes by looking ahead to more advanced physics and considering the prospects for interpreting quantum field theory and general relativity as theories of dynamic production.

2 The Time Evolution Paradigm

In this section, we will work our way to a careful presentation of the time evolution paradigm. The idea that laws of nature dictate how the universe evolves over time may seem old-fashioned. Adlam (2022a) writes:

> "Newton bequeathed to us a picture of physics in which the fundamental role of laws is to give rise to time evolution: the Newtonian universe can be regarded as something like a computer which takes in an initial state and evolves it forward in time ... But science has come a long way since the time of Newton, and thus we should not necessarily expect that accounts of lawhood based on a Newtonian time-evolution picture will be well-suited to the realities of modern physics." (Adlam, 2022a, pg. 3)

Physics has indeed come a long way since Newton, but one can debate whether that progress has occurred within—or gone beyond—"the Newtonian time-evolution picture."

Adlam is not alone in presenting the time evolution paradigm as behind the times. The 2023 Foundational Questions Institute (FQxI) essay contest prompt began with this sentence:

> "Galileo claimed that the book of nature is written in mathematics, and indeed the discipline that he, Newton, and other 17th-century natural philosophers and mathematicians founded took on a particular form: mathematical laws expressing necessary relations between elements of the world, largely expressed in differential equations governing the time evolution of the state of the world." (FQxI, 2023)

The prompt later asks: "Could it have been otherwise? Should it be otherwise now?" The implication is that this is an outdated mold into which physical theories need no longer be cast. I take the conservative view that this mold should be retained and that more effort should be put into fitting existing and future theories into the mold. It has served physics well and should not be abandoned lightly.

Let us take a moment to breakdown the mold as described in the FQxI quotation. First, we have "mathematical laws" (the laws of nature), including "differential equations governing the time evolution of the state of the world" (the dynamical laws). Second, these laws express "necessary relations." This means that the laws do not merely describe what actually happens—they tell you what must happen. That is, they specify what is physically possible.

Third, the relations expressed by the laws must be between "elements of the world." Thus, the theory must specify what the elements of the world are—what exists, or, put another way, what the ontology of the theory is. Further, it is these elements of reality, and only these elements of reality, that should appear in the physical laws (Maudlin, 2018).

In Newton's physics, the laws are deterministic.[1] For a given past, they allow only a single future. It is sometimes said that the laws allow only a single future given the state at a moment, but here we must be careful. How does one specify the physical state at a moment? Albert (2000, pg. 9–10) gives two requirements for such a specification. First, the instantaneous state should describe features of that moment alone. Second, a full set of instantaneous states across all times should fully specify everything that occurs. You might initially think that the masses, positions, and velocities of bodies should all be included in the instantaneous state, but Albert uses his first requirement to disqualify velocity. Velocity is the rate at which position changes and the velocity at a moment can only be determined by considering (arbitrarily small) temporal neighborhoods around the moment in question. Rejecting velocity does not lead to a problem with the second requirement because once the positions are specified at each time the velocities are fixed. If we say that velocities are not included in instantaneous states, then the laws of Newton's physics will certainly fail to determine a unique future given the state at a moment. But, the laws will give a unique future given the state of the world over an arbitrarily short time interval. Let us thus follow Albert (2000, pg. 11) and Arntzenius (2000, pg. 195) and say that a theory is *deterministic* if and only if specifying the state of the world over an arbitrarily small time interval to the past of a given moment (assuming the laws to be obeyed during this interval) uniquely determines a single future that is allowed by the laws (a single future that is physically possible).[2] Although there are lines of reasoning that would lead you to expect that our laws will turn out to be deterministic, let us not attempt to settle the issue of determinism here. We can formulate the time evolution paradigm such that it also allows for stochastic laws. For *stochastic laws*, specifying the state of the world over an arbitrarily small time interval to the past of a given moment gives probabilities for a variety of different futures.

The velocity at a moment can be part of what determines the future of that moment, provided that we understand the relevant velocity to be the past velocity $\vec{v}^{\,p}$: a *past* derivative defined by considering arbitrarily small time intervals to the past of that moment,

$$\vec{v}^{\,p}(t) = \left(\frac{d}{dt}\right)^p \vec{x}(t) = \lim_{\delta\to 0} \frac{\vec{x}(t) - \vec{x}(t-\delta)}{\delta} \ , \tag{1}$$

as in Lange (2005, sec. 2). This velocity is depicted in figure 1. Modifying a proposal from Easwaran (2014), the acceleration that appears in the laws can be defined as the future

[1] Although the laws of Newtonian physics are generally presented as deterministic, there are in fact some subtle and difficult questions as to whether they are truly deterministic that we will not explore here (Earman, 2004; Wilson, 2009; van Strien, 2021).

[2] Builes & Impagnatiello (forthcoming) call this kind of determinism "Near Markovian Determination."

acceleration $\vec{a}^{pf}$: a *future* derivative of the past velocity,

$$\vec{a}^{pf}(t) = \left(\frac{d}{dt}\right)^f \vec{v}^p(t) = \lim_{\epsilon \to 0} \frac{\vec{v}^p(t+\epsilon) - \vec{v}^p(t)}{\epsilon} \ . \tag{2}$$

Thus, in Newton's second law the acceleration of a body can be understood as a future effect of present forces: $\vec{F} = m\vec{a}^{pf}$. This acceleration is depicted in figure 2.

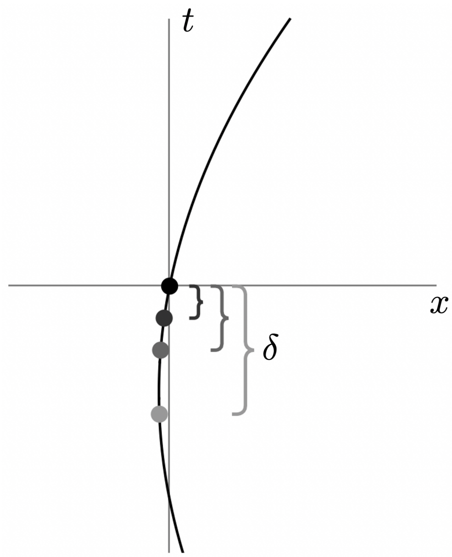

Figure 1: The past velocity $\vec{v}^p$ (1) of a body is determined by comparing the body's location at a moment to its location at progressively closer past moments (shown here as darkening dots).

Easwaran (2014) uses clever, but more complicated, definitions of the past and future time derivatives. These definitions avoid referencing the present moment by either considering two arbitrarily close past times (for the past time derivative) or two arbitrarily close future times (for the future derivative).[3] Easwaran calls these open-ended derivatives (because the endpoint at the present is left out) and contrasts them with the closed-ended derivatives defined above. Easwaran's choice to use open-ended derivatives allows him to classify past velocity as entirely about the past and future velocity as entirely about the future. To be more precise, he defines a past neighborhood property as follows:

> "A past neighbourhood property at t is a property of an object that is not grounded in the fundamental properties of the object at t, but, for every interval $\langle t-\Delta, t\rangle$, the fundamental properties of the object across that interval are sufficient to ground it." (Easwaran, 2014, pg. 849)

[3] In his notation, Easwaran (2014, pg. 849) defines the past velocity v_t^p (in one dimensional space) as the quantity that satisfies

$$\forall(\epsilon > 0)\exists(\delta > 0)\forall t', t'' \left((t - \delta < t', t'' < t) \to \left|\frac{x_{t'} - x_{t''}}{t' - t''} - v_t^p\right| < \epsilon\right) \ , \tag{3}$$

if there is such a quantity.

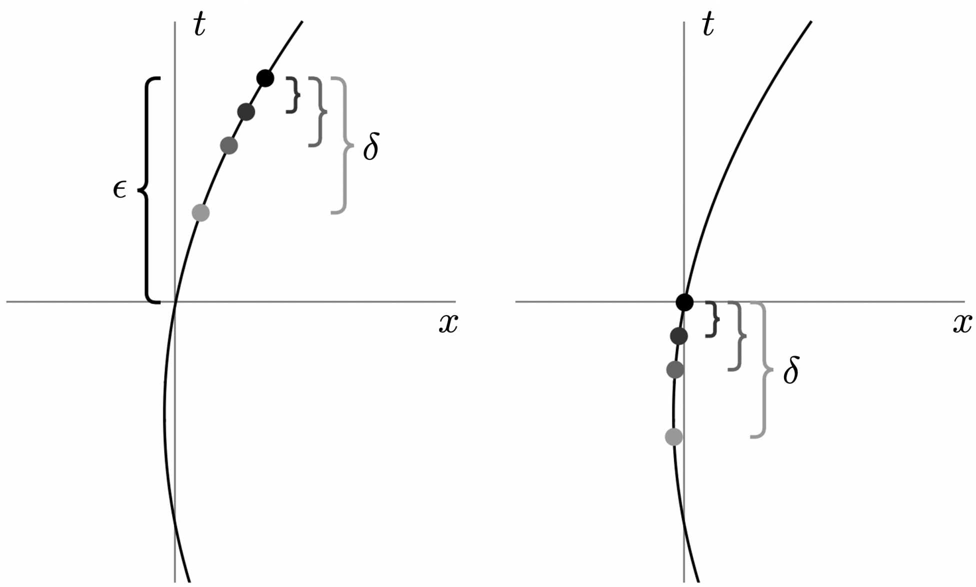

Figure 2: The future acceleration $\vec{a}^{pf}$ (2) of a body is determined by comparing (a) the body's past velocity at a time ϵ in the future (which is determined by looking δ into the past of that moment, as shown on the left), and (b) the body's past velocity at the moment in question (which is determined by looking δ into the past of that moment, as shown on the right). As is evident from the image on the right, a body's future acceleration $\vec{a}^{pf}$ depends on its past and is not a pure property of its present and future.

Future neighborhood properties are defined similarly, swapping $\langle t-\Delta, t\rangle$ for $\langle t, t+\Delta\rangle$. The angle brackets denote that the endpoints of these intervals are not included. For Easwaran, the acceleration that is defined as the future derivative of past velocity counts as a future neighborhood property because: for any time t' within the interval $\langle t, t+\Delta\rangle$ that you might look to for the future derivative, you can find arbitrarily close pairs of earlier times within the interval to apply his open-ended past time derivative and find the past velocity. Here we will adopt the simpler closed-ended time derivatives and accept that future acceleration is not a property of the future alone—as you might have desired for the output of a dynamical law—but instead describes how properties that look to the past change as you move to the future (figure 2). Because the past merely serves as a reference from which to evaluate changes toward the future, I still think it is sensible to regard the future acceleration $\vec{a}^{pf}$ from (2) as a future effect of present forces in Newton's second law. One reason behind the choice to use closed-ended derivatives will be given later when we see that, in electromagnetism, even if we were to use open-ended derivatives the laws would not give pure properties of the future light-cone as output.

Because our definition of past velocity depends on the present, let us adopt the following definition of a past neighborhood property (modifying Easwaran's definition to include the present moment):

> A **past neighborhood property** of an object at t is not determined by the present properties of the object at t alone, but is determined once the present properties of

the object are specified within any arbitrarily small interval $[t-\Delta, t]$.[4]

A future neighborhood property could be defined similarly, replacing $[t-\Delta, t]$ by $[t, t+\Delta]$. The square brackets denote that the endpoints of these intervals are included. The acceleration $\vec{a}^{\,pf}$ can be classified as a past-to-future neighborhood property:

> A **past-to-future neighborhood property** of an object at t is not determined by the present and past neighborhood properties of the object at t alone, but is determined once the present and past neighborhood properties of the object are specified within any arbitrarily small interval $[t, t+\Delta]$.

Note that, by this definition, future neighborhood properties also count as past-to-future neighborhood properties because they are determined by the present properties over an arbitrarily small interval to the future. However, some past-to-future properties—like the acceleration $\vec{a}^{\,pf}$— are not future neighborhood properties. If we require dynamic laws to give past-to-future neighborhood properties as output, we allow for either first-order dynamics (as in the Schrödinger equation) where the output is a future neighborhood property or second-order dynamics (as in Newton's second law) where the output is a past-to-future neighborhood property, but not a future neighborhood property.

To avoid action at a temporal distance, Easwaran (2014, pg. 857) conjectures that "The fundamental causal [dynamical] laws must use present properties and past neighbourhood properties to determine future neighbourhood properties." This idea has not been widely adopted, but it is an attractive proposal that fits well with a dynamic production account of laws. Let us take a modified version of the idea onboard here and say that a law counts as a *temporally local dynamical law* if it gives past-to-future neighborhood properties as output and takes as input only present properties or past neighborhood properties. Dynamical laws that are not temporally local might take as input properties at earlier or later times, as in the retarded action-at-a-distance and Wheeler-Feynman versions of electromagnetism that will be discussed in section 5.

Putting all the pieces together, we arrive at a strict and more precise version of the time evolution paradigm. Let us say that a set of laws fits **The Time Evolution Paradigm** for a specified ontology if and only if the following three conditions are met:

1. **Temporally Local Dynamical Laws:** A subset of the laws (the dynamical laws) give past-to-future neighborhood properties of the specified ontology at a moment as output and take as input only present properties or past neighborhood properties of the specified ontology at that moment. The dynamical laws may give either precise values for the past-to-future neighborhood properties properties or probability distributions over such values.

2. **Non-Dynamical Laws:** The laws that do not take the above dynamical form (the

[4] Because we need at least some of the past to determine the velocity (or, using other words, to serve as grounds for the velocity), but we do not need any particular past moment, there is a puzzle as to exactly what are the facts about position in virtue of which the velocity at a moment is what it is (Builes & Teitel, 2019).

non-dynamical laws) express relations between present properties or past neighborhood properties of the specified ontology at a moment.

3. **Deterministic or Stochastic:** Once a law-abiding history for the specified ontology over an arbitrarily small time interval to the past of a given moment has been fixed, the laws either uniquely fix a single future sequence of states or the laws yield a precise probability distribution over future sequences of states.

This formulation allows for non-dynamical laws in addition to dynamical laws. As a candidate example of a non-dynamical law, Maudlin (2007, pg. 13) gives Newton's law of gravitation, which uses present positions and masses to specify the present force on a given body. Maudlin describes this law as an "adjunct principle" that allows the dynamical law $\vec{F} = m\vec{a}$ to predict future time evolution. Chen & Goldstein (2022, pg. 60) consider the example of Gauss's law, $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$—which relates the present divergence of the electric field to the present distribution of charge. Although Chen & Goldstein (2022) and Chen (2023) portray Maudlin as requiring all laws to be dynamical laws, I take his written work to leave open the possibility that there may be non-dynamical laws (as is evident in Maudlin, 2007, pg. 13–14; Maudlin, 2018). In any case, I will not require that all laws be dynamical laws. That being said, the requirement in the second condition that other laws "express relations between present properties or past neighborhood properties" does rule out certain putative laws, such as those that posit probability distributions over initial conditions (Chen & Goldstein, 2022, sec. 3.3.3). The third condition, that the laws be either deterministic or stochastic, is meant to exclude laws that give incomplete stories about the future: neither determining the future uniquely nor giving definite probabilities for different futures.

Next, let us review the three competing philosophical accounts of laws of nature and how they each might view the time evolution paradigm. Then, we will see how classical mechanics and electromagnetism might be fit into the time evolution paradigm and discuss updating the paradigm in a relativistic context.

3 Three Accounts of Laws of Nature

Philosophers disagree as to whether the laws of nature truly govern what happens in nature or whether they merely describe patterns in what happens. Chen & Goldstein (2022) call this "the great divide." If the laws of physics simply describe patterns, then the laws are not fundamental constituents of our universe—they emerge from an analysis of the more fundamental history of all the things that happen in the universe throughout its past and future. The laws can be reduced to the history of the universe. By contrast, laws that truly govern may be basic parts of the universe—in addition to the particles, fields, or whatever else features in the ontology of the final theory. That kind of view is called primitivist because the laws cannot be reduced to anything more fundamental. In this section we will analyze three philosophical accounts of laws of nature: one reductionist account that falls on the laws-describe side of the great divide (the best systems account) and two primitivist accounts that fall on the laws-govern side of the divide (the dynamic production and laws-as-constraints accounts). Adlam (2022a), Chen & Goldstein (2022), and Chen (2023) survey a few other accounts—the universals account (Platonic reductionism), the powers account (Aristotelian reductionism), and the modal structure account—that will be set aside here to focus our discussion.

Let us begin with the best systems account, originally put forward by David Lewis and prominently defended by Barry Loewer.[5] On this account, what is fundamental is the history of the universe—a specification of what is happening at every point in spacetime. That specification is supposed to be done using only certain special properties called "Humean properties" (after the 18$^{\text{th}}$ century philosopher David Hume), explained by Loewer as follows:

> "Call a property 'Humean' if its instantiation requires no more than a spatiotemporal point and its instantiation has no metaphysical implications concerning the instantiations of fundamental properties elsewhere and elsewhen. Lewis's examples of Humean properties are the values of electromagnetic and gravitational fields and the presence or absence of a material particle at a point." (Loewer, 1996, pg. 102)

Specifying the Humean properties at every point in spacetime is supposed to give the entire history of the universe, called the "Humean Mosaic." The mosaic contains everything ever has or will happen, nearby or far away, observed or unobserved. Various candidate sets of laws of nature (called "systems") can be compared to see which does the best job at capturing patterns in the mosaic, traditionally determined by weighing three key virtues: strength, simplicity, and fit. Candidate systems are only allowed into the competition if they have no false consequences about the universe, and are considered stronger the more informative their true consequences are. Roughly, you might say that strength is a measure of how many possibilities are ruled out by a given system, though there are infinities at play that will make it hard to render this gloss precise. The goal is to find a strong system that is also relatively simple, as simplicity is a virtue that matters in addition to strength. Like strength, it is difficult to precisely measure simplicity. The third and final virtue is fit. If a system assigns probabilities for certain events to occur, one must consider how well those probabilities match reality. The higher the probability assigned to what actually happens, the better the system scores on fit. Ideally, one system will emerge as the clear winner when judged on strength, simplicity, and fit. This system is crowned as the best system and it gives the actual laws of nature. Of course, our ability as humans to know the details of the mosaic and judge competing systems is limited. But, the idea here is that there is a fact out there about what the best system would be for the mosaic as a whole and that our scientific explorations can potentially point us towards it.

The best systems account does not require the laws of nature to fit the time evolution paradigm from the previous section, but deterministic or stochastic temporally local dynamical laws do have a good shot at being included in the best system because they get you a lot from a little: allowing you to predict the future (at least probabilistically) from an arbitrarily thin time slice. Callender (2017, ch. 7-8) argues that time is a special dimension within the best systems

[5]For an introduction to the best systems account, see Loewer (1996); Lange (2008).

account because the laws that emerge as part of the best system include laws of time evolution. As Callender (2017, pg. 142) puts it, "Time is that direction on the manifold of events in which we can tell the strongest or most informative stories."

The best systems account can be criticized for failing to explain why things happen the way they do. The account starts with a history of things that happen and then seeks to find patterns in this history. By contrast, the dynamic production account views the history as generated by the laws. This account does not require the history of the universe to be a mosaic of Humean properties, but if we temporarily adopt the Humean mosaic language we can say that the mosaic is produced by the laws. Here is how Tim Maudlin, our primary proponent of dynamic production, puts it:

> "The universe started out in some particular initial state. The laws of temporal evolution operate, whether deterministically or stochastically, from that initial state to generate or produce later states. And the sum total of all the states so produced *is* the Humean Mosaic." (Maudlin, 2007, pg. 174)

It is unclear how seriously Maudlin takes the idea of an initial state—and in any case the contours of the dynamic production account discussed here need not match Maudlin's exactly—but let us not build into the account any assumption that there was a first moment.[6] The key point here is that the laws allow us to explain future states from past states using the laws of nature because the laws are additional features of the universe beyond the things within the universe that are governed by the laws.

The dynamic production account relies on a fundamental asymmetry between past and future (a fundamental arrow of time),[7] but Maudlin (2007, pg. 108–109) argues that it is compatible with the block universe theory according to which the past and future are just as real as the present. It would be worrisome if the account was not compatible with the block universe theory because special relativity arguably forces us to the block universe theory.[8] Dorst (unpublished) has challenged the compatibility, asking how the laws could possibly produce the future if it is already out there. This is a serious criticism that could be discussed in depth, but let me briefly point out that the a similar argument could be given against the possibility of any ordinary act of production within the block universe theory, such as the creation of a sculpture. How can Michelangelo create the sculpture of David if it is already out there (in the future)? One way of responding to Dorst would be to argue that just as ordinary acts of production (like Michelangelo's) are compatible with the block universe theory, the dynamic production of the

[6] Chen & Goldstein (2022, pg. 61) criticize the assumption of a first moment because it would rule out spacetimes without temporal boundaries. They say that it appears to be an important part of the dynamic production account because "it is what gets the entire productive enterprise started," but I do not see any problem with the idea that production has always been occurring no matter how far back you go in time. Given the articulation of the time evolution paradigm in section 2, it is actually not quite right to say that the first moment gets the productive enterprise started. We normally need an arbitrarily thin time slice preceding a given moment for the laws to generate time evolution (because we need velocities and other rates of change). Any moment after the first can be explained as the product of an earlier arbitrarily thin time slice evolving via the laws, but the first moment alone would be insufficient to get the evolution going.

[7] See Loewer (2012).

[8] See Putnam (1967); Zimmerman (2011).

laws is compatible with the block universe theory. While it is true that from an outside-of-time perspective the future "already" exists, production is something that happens within time and at a moment the future will exist but does not *already* exist.

The dynamic production account appears to require the laws of nature to fit something like the time evolution paradigm from section 2. The dynamical laws of that paradigm would be what Maudlin (2007) calls the fundamental laws of temporal evolution (FLOTEs). It is these laws that generate time evolution, guiding the ontology and producing future states from earlier ones. Chen & Goldstein (2022, pg. 46 & 60) challenge the idea of dynamic production because the state of the universe at a moment does not contain features like momentum that are needed to evolve the state forward via the laws. However, following our discussion from section 2, we can take momentum to be a past neighborhood property (like velocity) and understand the relation of dynamic production to connect the present, and its past neighborhood, to the future. The way that the dynamical laws of section 2 connect past to future also helps to address Loewer's (2012, pg. 133) remark that "there isn't a 'next' state if time is continuous" to support the claim that one state of the universe produces the next via the laws.

Non-dynamical laws play a quite different role from the dynamical laws, constraining the relations between properties of the present and arbitrarily-short past at a moment. Maudlin (2007, ch. 1) calls such laws adjunct principles and describes them as principles "that are needed to fill out the FLOTEs in particular contexts, principles about the magnitudes of forces and the form of the Hamiltonian, or about the sorts of physical states that are allowable." As was mentioned earlier, I will understand the dynamic production account as allowing for such non-dynamical laws that supplement the dynamical laws. The non-dynamical laws restrict the starting points from which dynamic production might occur.

The third condition of the time evolution paradigm requires the laws to be either deterministic or stochastic. This is a natural requirement for the dynamic production account. To dynamically produce the future, the laws must be sufficiently specific that they either yield a unique future or a probability distribution over possible futures. If the laws give probabilities, those probabilities should be understood as basic propensities that are not to be reduced to anything more fundamental, such as long-run frequencies (Frigg & Hoefer, 2007; Loewer, 2012, pg. 118).

A defender of dynamic production need not claim that it is metaphysically impossible to have laws of nature that violate the time evolution paradigm and cannot be understood as dynamically producing the future from the past. Instead, they could simply claim that the laws of our world play this special metaphysical role. Dynamic production is not part of what it takes for something to be a law, it is merely a feature of our laws. It is difficult to say what it takes in general for something to be a law, but you might say that at a minimum it must constrain what is physically possible. Here the dynamic production account can make peace with the next account to be discussed: what I will call the "laws-as-constraints account," putting an umbrella over what Chen & Goldstein (2022) call "minimal primitivism" and Adlam (2022a,b) calls "the constraint framework." The dynamic production account sticks its neck out further and we can debate whether that boldness is wise. Proponents of the laws-as-constraints account prefer the flexibility of their account, arguing that the laws of our world may well turn out not to include dynamical laws and that our philosophical theorizing should not preclude this possibility. Proponents of dynamic production can respond that we have good reason to expect the final laws to include laws of time evolution because our most successful extant theories are theories of time evolution, or at least can be formulated as theories of time evolution.

Chen & Goldstein (2022, pg. 23) present their account as follows:

> "On our view, fundamental laws govern by constraining the physical possibilities of the entire spacetime and its contents. They need not exclusively be dynamical laws, and their governance does not presuppose a fundamental direction of time."

Like the dynamic production account, the laws-as-constraints account does not have to view the history of the universe as a mosaic of Humean properties, but adopting the mosaic language the view can be put succinctly as: "the laws of nature are understood to take the form of sets of Humean mosaics, or probability distributions over each set" (Adlam, 2022a, pg. 25). The laws of nature impose constraints that restrict the physically possible histories of the universe, leaving a set of allowed histories. We will take this to be a primitivist account where the laws are not reduced to anything more fundamental, though Adlam (2022a, sec. 3.3) leaves the door open for a future reduction. Probabilistic laws might be accommodated as Adlam suggests by having the laws assign probabilities to histories or in some other way. Chen & Goldstein (2022, sec. 3.3.3) survey five options and Barrett & Chen (2023) further explore Chen & Goldstein's fourth option, according to which the laws simply select a set of allowed histories—roughly put, these would be histories that match what you would expect to see if certain probabilistic laws were in place.

The laws-as-constraints account does not require the laws of nature to fit the time evolution paradigm from section 2, but it is compatible with a preference for dynamical laws. As Chen & Goldstein (2022, pg. 50) write, "The preference for FLOTEs and dynamical laws more generally may be explained by a preference for laws that strike a good balance between simplicity and informativeness." The laws are not determined by a competition of strength and simplicity (as in a Humean account), but these factors can still serve as guides as we seek to determine the laws of nature from the evidence that we have.

Although the laws-as-constraints account permits laws that fit the time evolution paradigm, one of the main motivations for the view is a skepticism as to whether the final laws of our world will indeed fit the time evolution paradigm. This debate hinges on difficult questions about the interpretation of general relativity and quantum field theory that will only be discussed briefly in the conclusion. In this paper, I would like to focus on theories that seem like they should fit the time evolution paradigm to better understand how dynamic production works in these cases. Figuring out how the laws might produce the future in these easier cases lays the groundwork for tackling the harder cases.

4 Classical Mechanics

In classical mechanics with point particles, you might say that there is just one dynamical law: Newton's second law, force equals mass times acceleration. Following Easwaran (2014), as discussed in section 2, we can understand this law as taking the net force $\vec{F}_i$ on a given mass m_i as input and giving the body's future acceleration (2) as output,

$$\vec{F}_i = m_i \vec{a}_i^{\,pf} \quad \text{(dynamical law)} \ . \tag{4}$$

For this to be a dynamical law fitting the time evolution paradigm, the force would have to be a present or past neighborhood property. If the only forces at play are gravitational forces, then the net force on a mass is a present property set by Newton's law of gravitation,

$$\vec{F}_i = \sum_{j \neq i} -G \frac{m_i m_j}{r_{ji}^2} \hat{r}_{ji} \quad \text{(non-dynamical law)} \ . \tag{5}$$

Here $\hat{r}_{ji}$ is the unit vector pointing from m_j to m_i and r_{ji} is the distance between m_j and m_i. Maudlin (2007, pg. 13) describes Newton's law of gravitation as a non-dynamical law—an adjunct principle. Including it as such would yield a set of laws that fit the time evolution paradigm. In this case, it is straightforward to combine the dynamical and non-dynamical laws to get a single dynamical law that can govern on its own (without the need for any non-dynamical laws),

$$\vec{a}_i^{\,pf} = \sum_{j \neq i} -G \frac{m_j}{r_{ji}^2} \hat{r}_{ji} \quad \text{(dynamical law)} \ . \tag{6}$$

We can thus formulate Newtonian gravity either as a theory with two laws (one dynamical and one not), (4) and (5), or as a theory with a single dynamical law, (6).

One might object to the earlier characterization of Newton's second law in terms of future acceleration (4). That law is time-asymmetric, but Newtonian gravity is ordinarily taken to be a time-symmetric theory. Let me make two remarks about this. First, although the ordinary math may not be time-asymmetric, explanations of the math generally are. Forces are described as causing future acceleration, an understanding that fits well with (4) (Easwaran, 2014, pg. 852–853). Second, the theory remains time-symmetric in the sense that it is time-reversal invariant: for any history that obeys (4) and (5), the time-reversed history will obey (4) and (5). The key point here is that any history where $\vec{F} = m\vec{a}^{\,pf}$ is also a history where $\vec{F} = m\vec{a}^{\,p}$ because $\vec{a}^{\,pf}$ cannot differ from $\vec{a}^{\,p}$ if particles are only interacting through well-behaved gravitational forces.

The above story about the laws of Newtonian gravity changes if you take the ontology to include a gravitational field $\vec{g}$ in addition to the point masses. Then, you might try the following collection of laws:

$$\begin{aligned} \vec{F}_i &= m_i \vec{a}_i^{pf} && \text{(dynamical law)} \\ \vec{F}_i &= m_i \vec{g}(\vec{x}_i) && \text{(non-dynamical law)} \\ \vec{g}(\vec{x}) &= \sum_j -G\frac{m_j}{|\vec{x}-\vec{x}_j|^2} && \text{(non-dynamical law)}\ . \end{aligned} \tag{7}$$

The first law is Newton's second law, appearing again as the only dynamical law. The second law relates the force on a body to the gravitational field at the body's location $\vec{x}_i$. The third law gives the gravitational field resulting from a certain arrangement of masses. It is interesting to note that the gravitational field does not have its evolution determined by a dynamical law. It is rebuilt at each moment by a non-dynamical law.

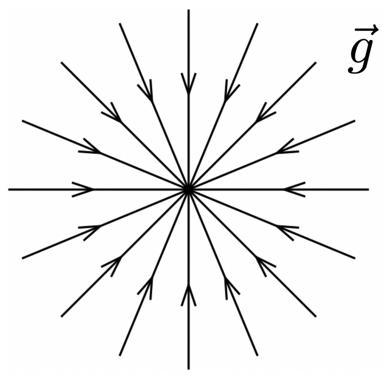

Figure 3: The gravitational field of a point mass is ill-defined at the location of that point mass.

The set of laws for Newtonian gravity (including a gravitational field) that were just given (7) will not function properly because they have problems with self-interaction. As you approach the location of any point mass, the gravitational field becomes infinitely strong and does not have a clear direction (figure 3). The field is ill-defined at the mass's location, which is exactly where you would like to use it to calculate the force on the mass. One way to address this problem is to introduce a separate gravitational field sourced by each particle and stipulate that particles do not feel their own fields. Then you could derive Newton's law of gravitation (5) with the important $j \neq i$ in the sum. A different strategy for addressing self-interaction would be to retain a single gravitational field and replace point masses by extended bodies, either rigid bodies or continua.[9] Then, we would have laws like:

$$\vec{\nabla} \cdot \vec{g} = -4\pi G \rho^m \quad \text{(non-dynamical law)} \tag{8}$$

$$\vec{\nabla} \times \vec{g} = 0 \quad \text{(non-dynamical law)} \tag{9}$$

$$\vec{f} = \rho^m \vec{g} \quad \text{(non-dynamical law)}\ , \tag{10}$$

[9]For discussion of the different possible ontologies for classical mechanics (point particles, rigid bodies, and continua), see Wilson (1998, 2013); van Strien (2021).

where G is the gravitational constant, ρ^m is the mass density, and $\vec{f}$ is the density of force exerted by the gravitational field on matter. (Note that because Newtonian gravity closely parallels electrostatics, the first two laws resemble Maxwell's equations as they apply to electrostatics.) None of the three laws presented above are dynamical laws. To get dynamics, we would also need a version of Newton's second law. For rigid bodies, one can integrate the force density over the volume of the body to get a net force and use this to determine how the center of mass moves. One would also need a law connecting the density of force on the body to its rotation. For continua, the reaction to experienced forces will depend on the nature of the body experiencing the forces. (We'll return to that issue in section 7.) Eliminating the gravitational field, you could alternatively have rigid bodies or continua interacting by gravitational action-at-a-distance—as in (5). All of this begins to illustrate the plethora of options that you have for understanding the laws and ontology of classical mechanics while staying within the time evolution paradigm.

Although there are many ways to formulate laws of classical mechanics that fit within the time evolution paradigm, there are ways to formulate the laws that appear to break with this paradigm. Adlam (2022a, pg. 3–4, 35) and Chen & Goldstein (2022, pg. 46–47) cite the Lagrangian stationary action approach to classical mechanics, where one can determine whether potential histories for a system are allowed or forbidden by seeing whether they extremize (either maximize or minimize) the action functional (a path integral of the Lagrangian evaluated along the system's trajectory through its configuration space). As this method takes the history as input and returns either allowed or forbidden as output, it does not seem to fit the time evolution paradigm. Chen & Goldstein (2022) acknowledge that one can get the same division between allowed and forbidden histories from laws of time evolution, but they take it to be an advantage of their laws-as-constraints account that they permit the Lagrangian method to directly give a candidate law of nature. Adlam (2022a, pg. 3) writes that "Lagrangian methods are well-recognised as valuable mathematical tools, but they have not usually been taken seriously as possible descriptions of reality, presumably because the Lagrangian is optimized over an entire history and thus the Lagrangian description can't straightforwardly be understood within the standard time-evolution picture." It is fair for these authors to note that classical mechanics does not need to be formulated as a theory of time evolution. My aim in this section has been to canvas some of the ways that it can be formulated as a theory of time evolution so that we can begin to confront the challenges facing dynamic production. Let us now proceed to more advanced physics: classical electromagnetism.

5 Electromagnetism, First Pass

To formulate the laws of electromagnetism, we must answer a central question about the theory's ontology: Does the electromagnetic field really exist or is it merely a tool that can be used to calculate the forces between bits of charged matter? One way to eliminate the electromagnetic field is to formulate electromagnetism as a theory of retarded action-at-a-distance.[10] The force

[10]See Lange (2002); Frisch & Pietsch (2016); Hubert & Sebens (2023).

on a given particle now is the result of interactions between that particle now and other particles at earlier times—the times when their world-lines intersect the past light-cone of the given particle. This version of electromagnetism breaks the time evolution paradigm because the interactions are not temporally local. Another way to eliminate the electromagnetic field is to include half-retarded half-advanced action-at-a-distance, as in the Wheeler-Feynman version of the theory. The force on a given particle now is the result of interactions with other particles intersecting both the past and future light-cones of that particle. Again, we have a theory that violates the time evolution paradigm because it is not temporally local. Chen & Goldstein (2022) give Wheeler-Feynman electromagnetism as an example of a theory that their minimal laws-as-constraints account can accommodate and a dynamic production account cannot.

Although the retarded action-at-a-distance and Wheeler-Feynman version of electromagnetism are worthy of study, there are good reasons to prefer a version of electromagnetism that takes the electromagnetic field to be real. A real electromagnetic field can ensure conservation of energy and momentum by itself possessing energy and momentum.[11] When we look to ahead to quantum field theory, the electromagnetic field is treated in a similar way to charged matter and thus it is natural (with the benefit of hindsight) to view both as real in classical electromagnetism (Sebens, 2022a; Hubert & Sebens, 2023).[12]

Accepting the electromagnetic field as real, one can still debate whether the electromagnetic field's behavior can always be traced back to past sources or whether the field has independent degrees of freedom. Put more precisely, one can debate whether the electromagnetic field must obey a radiation condition restricting it to be the kind of electromagnetic field that you might introduce as a calculational tool within a retarded action-at-a-distance version of electromagnetism. If you do impose such a restriction, the electric and magnetic fields at a given point in space and time can be determined by looking at the behavior of charged matter along the past light-cone of that point via Jefimenko's equations (Griffiths, 2013, sec. 10.2.2). Jefimenko (2000, ch. 1) criticizes Maxwell's equations because none fit the following mold:

> "...equations depicting causal relations between physical phenomena must, in general, be equations where a present-time quantity (the effect) relates to one or more quantities (causes) that existed at some previous time." (Jefimenko, 2000, pg. 4)

Jefimenko takes Maxwell's equations to express relations between present-time quantities. (We will see shortly that, by interpreting the time derivatives as future derivatives, two of Maxwell's equation can actually be understood as linking cause and effect with cause preceding effect.[13]) Unlike Maxwell's equations, Jefimenko's equations connect present fields to past sources and

[11] Lazarovici (2018, sec. 4.2) gives a response to this concern defending the Wheeler-Feynman theory. Lange (2002) presents the argument for the reality of the electromagnetic field from conservation of energy and momentum, but finds it lacking in a relativistic context because energy and momentum are frame-dependent quantities. He bases his argument for the field's reality on the fact that it possesses mass.

[12] Proponents of the Wheeler-Feynman theory can of course pursue a revisionary approach to quantum field theory that eliminates the electromagnetic field and thus does not treat it in a similar way to charged matter.

[13] This response to Jefimenko is similar to the response that Easwaran (2014, pg. 853) gives to Field's (2003, pg. 438) general claim that the differential equations that appear in laws of physics cannot be interpreted causally because they relate quantities at the same time.

thus straightforwardly fit his mold. However, that connection is temporally non-local and thus, although they may conform to our basic expectation that causes should come before effects, Jefimenko's equations do not fit the time evolution paradigm as stated in section 2.

The restricted version of electromagnetism that Griffiths and Jefimenko prefer forbids unsourced radiation. It thus appears to yield a natural explanation as to why electromagnetic waves diverge—an explanation of the arrow of electromagnetic radiation. That asymmetry can also be explained in an unrestricted version of electromagnetism, as it is improbable that any unsourced contribution to the electromagnetic field would result in coordinated converging waves (North, 2003; Hubert & Sebens, 2023). There is room to debate which version of electromagnetism gives a better explanation of this asymmetry, but I prefer the explanation offered by the unrestricted version because the same kind of explanation can be given as to why entropy increases and why other waves diverge. It also fits well with quantum field theory, where the quantum electromagnetic field is not restricted to always have past sources (where, put another way, photons no more need past sources than electrons do).[14]

Having focused our attention on versions of electromagnetism that treat the electromagnetic field as real and do not impose a radiation condition requiring that field be traceable to past sources, there is still plenty of room to consider different versions of the theory. Maudlin (2018) presents 10 distinct proposals for the laws and ontology. It is not hard to find a version of the theory that fits the time evolution paradigm. We can start with the standard combination of Maxwell's equations and the Lorentz force law,[15] written here in Gaussian units:

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \tag{11}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{12}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{13}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{14}$$

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right) . \tag{15}$$

If we understand the time derivatives in (12) and (14) to be future time derivatives, then those two equations can be dynamical laws for the electric and magnetic fields.[16] The other three laws would be non-dynamical laws. To determine the future time evolution, the Lorentz force law (15) would need to be accompanied by a dynamical law for particle motion, like Newton's second law (4) (modified to be made relativistic, as will be discussed in due course). Note that here we are adopting an ontology of point particles for the charged matter and thus the charge and current densities will feature delta functions (as in Landau & Lifshitz, 1971, sec. 28).

[14] See Wald (2022, sec. 1.3); Hubert & Sebens (2023, sec. 3.3).

[15] See, e.g., Feynman *et al.* (1964, table 18-1).

[16] Easwaran (2014, pg. 857) gives a similar analysis of the Schrödinger equation: "In quantum mechanics, the Schrödinger equation directly sets the first derivative of the quantum wave-function, which must thus be a future neighbourhood property."

The use of point particles will lead to trouble with self-interaction because the electromagnetic field will become infinitely strong at the location of any particle (just where the field's value matters for calculating forces). I will set these problems aside for now and we will return to them in section 7. To remove infinities, it would be better to work with charge distributions and avoid point charges.

6 The Relativistic Time Evolution Paradigm

Although it is not difficult to find a version of electromagnetism that fits the time evolution paradigm, that paradigm may be too loose. Electromagnetism is a relativistic theory and as such we can put tighter constraints on the form that dynamical laws can take. They must be spatiotemporally local,[17] not merely temporally local. Here is how Maudlin (2007, pg. 60) puts the requirement of relativistic locality: "the physical state at any point of space-time is determined or influenced only by events in its past light-cone."[18] If the state at any point can only be influenced by events in its past light-cone, then the state at a particular space-time point can only influence events in the future light-cone of that point (for these are the events that it is in the past light-cone of). Given Bell's theorem, quantum physics may force us to give up on this kind of locality (Maudlin, 2014). But, it seems like it should at least be achievable within classical electromagnetism.

Instead of connecting present and past neighborhood properties at a time to past-to-future neighborhood properties, one might expect relativistic dynamical laws to connect properties at a space-time point and past light-cone neighborhood properties to past-to-future light-cone neighborhood properties. Those terms can be defined as follows (modifying the definitions from section 2):

> A **past light-cone neighborhood property** of an object at $(\vec{x}, t)$ is not determined by the space-time point properties of the object at $(\vec{x}, t)$ alone, but is determined once the space-time point properties of the object are specified within the past light-cone of $(\vec{x}, t)$ over any arbitrarily small interval $[t - \Delta, t]$.

> A **past-to-future light-cone neighborhood property** of an object at $(\vec{x}, t)$ is not determined by the space-time point and past light-cone neighborhood properties of the object at $(\vec{x}, t)$ alone, but is determined once the space-time point and past light-cone neighborhood properties of the object are specified within the future light-cone of $(\vec{x}, t)$ over any arbitrarily small interval $[t, t + \Delta]$.

One could of course also define future light-cone neighborhood property in parallel to past light-cone neighborhood property. As before, it will follow from the definitions that future light-cone neighborhood properties are past-to-future light-cone neighborhood properties.

[17] Here I am talking about spatiotemporal locality in the special relativistic sense, not the broader sense in which there is no light-speed limit on influences and the only constraint is that there be spatiotemporal continuity (see Lange, 2002, ch. 1).

[18] See also Maudlin (2007, pg. 20 & 30).

With these definitions in hand, let us say that a set of laws fits **The Relativistic Time Evolution Paradigm** for a specified ontology if and only if the following three conditions are met (with changes from the less strict time evolution paradigm of section 2 marked in italics):

1. ***Spatiotemporally* Local Dynamical Laws:** A subset of the laws (the dynamical laws) give past-to-future *light-cone* neighborhood properties of the specified ontology at a *point in space-time* as output and take as input only *space-time point* properties or past *light-cone* neighborhood properties of the specified ontology *at that space-time point.* The dynamical laws may give either precise values for the past-to-future *light-cone* neighborhood properties properties or probability distributions over such values.

2. **Non-Dynamical Laws:** The laws that do not take the above dynamical form (the non-dynamical laws) express relations between *space-time point* properties or past *light-cone* neighborhood properties of the specified ontology *at that space-time point.*

3. **Deterministic or Stochastic:** Once a law-abiding history for the specified ontology over an arbitrarily small time interval to the past of a given moment has been fixed *(in some inertial reference frame)*, the laws either uniquely fix a single future sequence of states or the laws yield a precise probability distribution over future sequences of states.

Present properties (that make no reference to other times) have been replaced by space-time point properties (that make no reference to other times or places). Past neighborhood properties have been replaced by past light-cone neighborhood properties (that are defined in terms of arbitrarily small subregions of the past light-cone). Similarly, past-to-future properties have been replaced by past-to-future light-cone properties. The third condition acknowledges that the selection of a thin time slice will depend on the frame of reference and requires deterministic or stochastic dynamics going forward from any such slice. The three conditions are not meant to specify what it takes for a theory to be relativistic or to impose the strictest possible standards on relativistic laws. The conditions are just meant to give a baseline as to the way that you might expect time evolution to work in a relativistic theory that includes laws of time evolution. In particular, the third condition could be strengthened to demand a more local form of determinism or stochasticity. In a relativistic theory, specifying what is going on at a time within any spherical region (and its arbitrarily-short past light-cone) should either fix, or give probabilities for, future slices of the converging light-cone that the spherical region forms the base of.[19]

Dorst (unpublished) has criticized the dynamic production account of laws because it claims that earlier states produce later ones and this appears to require a preferred foliation—that is, a preferred way of carving space-time into simultaneity slices so that we can identify which slices are producing which. Chen & Goldstein (2022, pg. 46) give a quick version of this criticism as well. Looking at Bell's theorem in quantum physics, one might argue that we have independent reason to believe in a preferred foliation and be sanguine about its use here (Maudlin, 2007, pg. 117; Maudlin, 2018, sec. 6). The relativistic time evolution paradigm provides an alternative

[19]The source-free homogeneous wave equation achieves this kind of determinism, as is proved in Strauss (2008, sec. 9.1) and will be discussed in section 7.

way to respond to Dorst's worry. We can view the dynamic laws as producing the future at each space-time point, with the inputs lying within the past-light-cone and the outputs concerning the future light-cone. We can then collect these instances of production at-a-point to get production from one time slice to the next. Different foliations will correspond to different ways of collecting these instances of production.

I believe that Dorst would categorize this response as what he calls "pluralistic production," because for every foliation it is true that the earlier states produce the later states. Dorst rejects this response because if we ask what produces a given event e at some particular space-time point, we get too many answers—overdetermination. Can it really be that the previous time slices approaching the event on one foliation produce the event *and* the previous time slices approaching the event on another foliation produce the event when these time slices are so different? Yes, it can be so because in a relativistic theory it is only the points that are shared between these series that produce e. It is the points in the past light-cone of e that produce e. There are different ways to take a limit within this light-cone, but they capture the same chains of production (see figure 4).

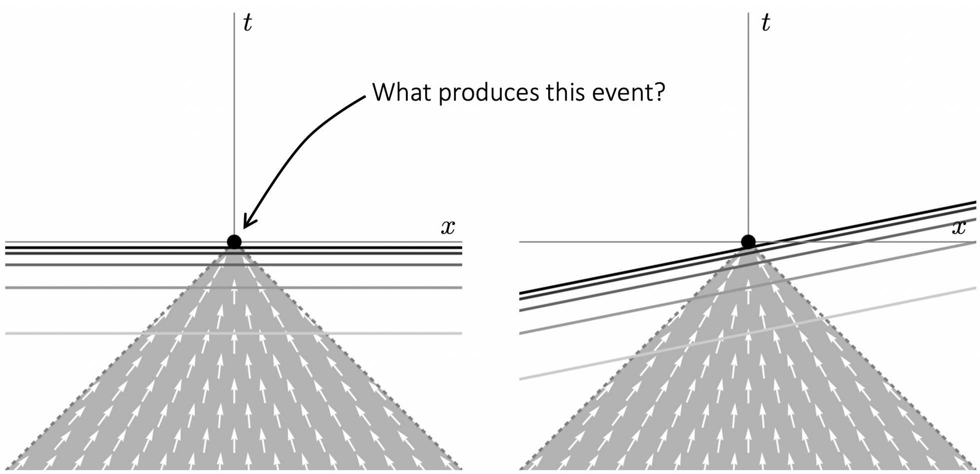

Figure 4: This figure shows schematically the chains of production connecting points in some event e's past light-cone to the event. It also shows two series of time slices approaching the event e that can be described as producing the event. The points in the event's past light-cone are carved up differently by the different series, but the underlying relations of production remain unchanged.

7 Electromagnetism, Second Pass

It requires a bit of work to fit electromagnetism into the relativistic time evolution paradigm, but the trick can be done. Here I present one way to do so, without claiming that it is the best and final way. We will need to find suitable laws governing the evolution of the electromagnetic field, the forces on charged matter, and the reaction of matter to forces. Let us begin with the field. We can use the scalar potential ϕ and the vector potential $\vec{A}$ to specify the state of the electromagnetic field, where these potentials are related to $\vec{E}$ and $\vec{B}$ by

$$\begin{aligned}\vec{E} &= -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\\ \vec{B} &= \vec{\nabla}\times\vec{A}\ .\end{aligned} \tag{16}$$

These relations ensure that two of Maxwell's equations are satisfied automatically, (12) and (13). Let us adopt the Lorenz gauge condition as a way of partially fixing the gauge freedom in the potentials,[20]

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0\ . \tag{17}$$

The remaining two of Maxwell's equations yield wave equations for ϕ and $\vec{A}$,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = -4\pi\rho \tag{18}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\frac{4\pi}{c}\vec{J}\ , \tag{19}$$

which can be more compactly using the d'Alembertian, $\Box = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$, as

$$\Box\phi = -4\pi\rho \tag{20}$$

$$\Box\vec{A} = -\frac{4\pi}{c}\vec{J}\ . \tag{21}$$

A first try at formulating these wave equations as laws of time evolution for the potentials, modeled on Easwaran's version of Newton's second law (4), would be to interpret the second derivative with respect to time as a future derivative of a past derivative (just as acceleration wast taken to be the future derivative of the past derivative of position),

$$\left(\frac{\partial}{\partial t}\right)^f\left(\frac{\partial}{\partial t}\right)^p\phi = 4\pi c^2\rho - c^2\nabla^2\phi \tag{22}$$

$$\left(\frac{\partial}{\partial t}\right)^f\left(\frac{\partial}{\partial t}\right)^p\vec{A} = 4\pi c\vec{J} - c^2\nabla^2\vec{A}\ . \tag{23}$$

These equations would yield temporally local but not spatiotemporally local dynamics. The ∇^2 operators yield neighborhood properties of the potentials that are determined by considering arbitrarily small spatial neighborhoods at the moment in question. That is forbidden by the relativistic time evolution paradigm. These are not past light-cone neighborhood properties.

Although this first attempt fails, we should be optimistic that the wave equations in (18) and (19) can be interpreted as giving spatiotemporally local dynamics. When the homogenous (source-free) wave equation, $\Box u = 0$, is discussed in textbooks on partial differential equations, it is standard practice to prove a "causality theorem" showing that a given spacetime point

[20] Adopting the Coulomb gauge would lead to very different laws for the potentials, laws that appear to involve instantaneous action-at-a-distance and to rely on a preferred simultaneity slicing (Maudlin, 2018). Such laws could be fit into the time evolution paradigm, but not the relativistic time evolution paradigm.

cannot influence anything outside its future light-cone and cannot be influenced by anything outside its past light-cone.[21] To be more precise, you can prove that if two solutions agree on the values of the function u and its time derivatives $\frac{\partial u}{\partial t}$ at a given time within the spherical spatial region bounded by the past light-cone of some future spacetime point $(\vec{x}, t)$, the two solutions will agree on the value of u at $(\vec{x}, t)$. Solutions to the inhomogeneous (sourced) wave equation, $\Box u = f$, will also satisfy such a causality theorem for a fixed source f because the any solution can be divided into a free part u_{in} (that obeys the homogenous wave equation and can be interpreted as describing incoming waves that are not attributable to the source function f) and a retarded part u_{ret} (where the value of u_{ret} at any space-time point is fixed by f along the past light-cone of that point). Given these causality results, it seems like there should be a way to formulate inhomogeneous wave equations, like the wave equations for the potentials (18) and (19), that makes the causal structure manifest and satisfies the conditions that we laid out earlier for spatiotemporally local dynamics linking past light-cone to future light-cone. As we will see, this can be done straightforwardly in one dimensional space, but is surprisingly difficult in three dimensional space.

Let us begin by considering the inhomogeneous wave equation in one dimensional space,

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) u = f \ , \tag{24}$$

with f treated as a fixed source function defined across space and time. D'Alembert's method for solving the homogeneous wave equation introduces new variables

$$\begin{aligned} \eta &= x + ct \\ \xi &= x - ct \ . \end{aligned} \tag{25}$$

We can use these variables to rewrite (24) as

$$4\frac{\partial}{\partial \eta}\frac{\partial}{\partial \xi} u = f \ , \tag{26}$$

where the derivatives are now evaluated along the two perpendicular edges of the light-cone.

Taking the first derivative in (26) to be a future derivative and the second to be a past derivative, the wave equation becomes

$$4\left(\frac{\partial}{\partial \eta}\right)^f \left(\frac{\partial}{\partial \xi}\right)^p u = f \ . \tag{27}$$

[21] For discussion of causality theorems for one and three-dimensional wave equations, see Zachmanoglou & Thoe (1976, ch. 8); Folland (1995, ch. 5); Evans (1998, sec. 2.4.3); Strauss (2008, sec. 9.1). For discussion of causality theorems in electromagnetism (making use of the Lorenz gauge), see Wald (1984, sec. 10.2); Wald (2022, sec. 5.4).

This can be written as a limit,

$$\lim_{\epsilon\to 0}\left(\lim_{\delta\to 0}\frac{u(x+c\epsilon+c\delta,t+\epsilon-\delta)-u(x+c\epsilon,t+\epsilon)-u(x+c\delta,t-\delta)+u(x,t)}{c^2\epsilon\delta}\right)=f\ . \tag{28}$$

This limit asks you to go up one edge of the light-cone shorter and shorter distances, each time taking a limit toward that edge of the light-cone from outside the light-cone in the direction perpendicular to the light-cone (as in figure 5). You are asking how the rate of change along one edge of the light-cone, approaching from the past, changes as you go up the other side of the light-cone into the future. This counts as a spatiotemporally local dynamical law because the left-hand side gives a past-to-future light-cone neighborhood property as output and the right-hand side gives what might be a space-time point property or a past light-cone neighborhood property, depending on the source function f (as we will see below). Note that the output is not a future-light-cone neighborhood property because at each point along the future light-cone that you consider for the derivative with respect to η, you must look outside the light-cone to evaluate $\left(\frac{\partial}{\partial\xi}\right)^p u$ (as is shown in figure 5). We can now follow up on a loose thread from section 2: Even if we were to follow Easwaran (2014) and use open-ended derivatives instead of closed-ended derivatives, the output of (27) would not be a future light-cone neighborhood property because the past derivative with respect to ξ takes you outside the future light-cone.

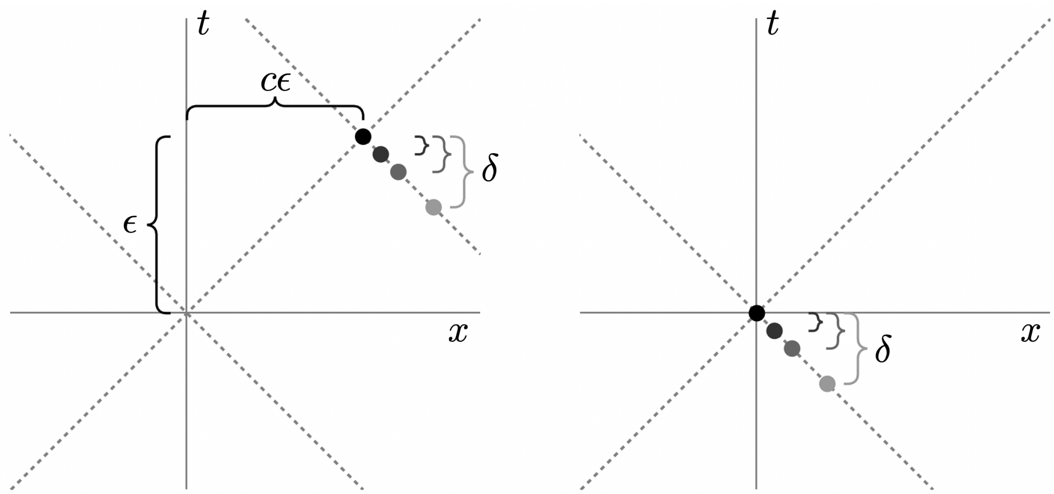

Figure 5: If we rewrite the one-dimensional wave equation using D'Alembert's variables as in (27) and (28), then one must compare (a) how the values of the function u change as you approach a spacetime point ϵ up the right edge of the future light-cone along the right edge of the past light-cone (left image) to (b) the way the values of u change as you approach the spacetime point of interest along the right edge of the past light-cone (right image).

Looking back at (27), you could just as well swap the order of the derivatives and go up the other edge of the future light-cone. To prepare for extension to three spatial dimensions, it would be best to imagine averaging the two orderings and not privileging either side of the future light-cone,

$$2\left(\frac{\partial}{\partial\eta}\right)^f\left(\frac{\partial}{\partial\xi}\right)^p u+2\left(\frac{\partial}{\partial\xi}\right)^f\left(\frac{\partial}{\partial\eta}\right)^p u=f\ . \tag{29}$$

We can take this to be our formulation of the one dimensional wave equation as a spatiotemporally local dynamical law.

Let us now consider the inhomogeneous wave equation in three dimensional space

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) u = f \ , \tag{30}$$

with f again treated as a fixed source function defined across space and time. The wave equations for the potentials, (18) and (19), fit this general form. Writing this wave equation as a spatiotemporally local dynamical law is not as straightforward as you might expect, but here is one way to do it. Let us focus our attention on the evolution at the origin, noting that any point in space can be treated as the origin. Using spherical coordinates with θ as the azimuthal angle and ϕ as the polar angle, the wave equation (30) for u at the origin becomes

$$\int \frac{d\theta d\phi}{4\pi} \sin\phi \left(3\frac{\partial^2}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) u = f \ , \tag{31}$$

where $\int \frac{d\theta d\phi}{4\pi}\sin\phi$, or $\int \frac{d\Omega}{4\pi}$, integrates over the solid angle Ω and divides by the total solid angle. For the time derivative term, this integral is trivial. To see that the Laplacian in (30) can be replaced by a radial second derivative in (31), plug the expansion

$$\frac{\partial}{\partial r} = \cos\theta\sin\phi\frac{\partial}{\partial x} + \sin\theta\sin\phi\frac{\partial}{\partial y} + \cos\phi\frac{\partial}{\partial z} \tag{32}$$

into the integral over the solid angle. The cross terms drop out and you are left with

$$\int \frac{d\Omega}{4\pi}\left(3\frac{\partial^2}{\partial r^2}\right) u = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) u = \nabla^2 u \ . \tag{33}$$

We can now introduce new variables going out along the future and past light-cones,

$$\begin{aligned} \eta &= r + ct \\ \xi &= r - ct \ , \end{aligned} \tag{34}$$

similar to D'Alembert's variables (25) from the one dimensional case. Using these variables, the wave equation at the origin (31) is

$$\int \frac{d\Omega}{4\pi}\left(2\frac{\partial^2}{\partial \eta^2} + 2\frac{\partial^2}{\partial \xi^2} + 8\frac{\partial^2}{\partial\eta\partial\xi}\right) u = f \ . \tag{35}$$

In the integral, for a given θ and ϕ, the $\frac{\partial^2}{\partial \eta^2}$ term asks you to take a second derivative along the future light-cone in that direction and the $\frac{\partial^2}{\partial \xi^2}$ term asks you to take a second derivative along the past light-cone in that direction. This past light-cone derivative should give the same result as taking the forward light-cone derivative in the opposite direction, $-\hat{r}$. Integrating over all angles,[22]

$$\int \frac{d\Omega}{4\pi} \left(\frac{\partial^2}{\partial \eta^2} \right) = \int \frac{d\Omega}{4\pi} \sin\phi \left(\frac{\partial^2}{\partial \xi^2} \right) \ . \tag{37}$$

We can thus combine the first two terms in (35) to simplify the equation, yielding

$$4 \int \frac{d\Omega}{4\pi} \left(\frac{\partial^2}{\partial \eta^2} + 2 \frac{\partial^2}{\partial \eta \partial \xi} \right) u = f \ . \tag{38}$$

Regrouping gives

$$4 \int \frac{d\Omega}{4\pi} \left(\frac{\partial}{\partial \eta} \left[\frac{\partial}{\partial \eta} + 2 \frac{\partial}{\partial \xi} \right] \right) u = f \ . \tag{39}$$

Let us now distinguish past and future derivatives as in (27),

$$4 \int \frac{d\Omega}{4\pi} \left(\left(\frac{\partial}{\partial \eta} \right)^f \left[\left(\frac{\partial}{\partial \eta} \right)^p + 2 \left(\frac{\partial}{\partial \xi} \right)^p \right] \right) u = f \ . \tag{40}$$

For given angles θ and ϕ, the integrand asks to consider how a sum of two past light-cone derivatives (one in the $\hat{r}$ direction and one in the $-\hat{r}$ direction) changes as you move along the future light-cone in the $\hat{r}$ direction (see figure 6). With this form of the wave equation, we have arrived at a spatiotemporally local dynamical law, with the left-hand side giving a past-to-future light-cone neighborhood property and the source function on the right-hand side being either a space-time point property or a past light-cone property (as we are about to see).[23]

The wave equation for the scalar potential (18) fits the form of a three dimensional wave equation (30) and thus we can formulate it as a spatiotemporally local law like (40),

$$\int \frac{d\Omega}{4\pi} \left(\left(\frac{\partial}{\partial \eta} \right)^f \left[\left(\frac{\partial}{\partial \eta} \right)^p + 2 \left(\frac{\partial}{\partial \xi} \right)^p \right] \right) \phi = -\pi \rho \ , \tag{42}$$

[22] Here is a more formal proof of (37). Expanded in terms of r and t derivatives, the difference between the left and right-hand sides of (37) is proportional to

$$\int \frac{d\Omega}{4\pi} \left(\frac{\partial}{\partial r} \frac{\partial}{\partial t} \right) = 0 \ . \tag{36}$$

The fact that this term vanishes can be seen by noting that radial derivatives in opposite directions will cancel, or, by using the expansion of $\frac{\partial}{\partial r}$ in (32).

[23] One should not read the speed of causal influence directly from the ability to put the wave equation in a spatiotemporally local form like (40). If you started with a three dimensional wave equation (30) with the speed of light of light doubled ($c \to 2c$), then causal influences would propagate in a way that violates a speed-of-light-based standard for spatiotemporal locality. However, you can follow parallel reasoning to the derivation above to get an expression for the wave equation at the origin as

$$\int \frac{d\Omega}{4\pi} \left(\left(\frac{\partial}{\partial \eta} \right)^f \left[\frac{11}{2} \left(\frac{\partial}{\partial \eta} \right)^p + \frac{13}{2} \left(\frac{\partial}{\partial \xi} \right)^p \right] \right) u = f \ , \tag{41}$$

which meets our earlier standard for being a spatiotemporally local dynamical law. We must reject this formulation as not accurately capturing the way that the past generates the future for the modified wave equation. [Note: Perhaps there is something better to say here. I welcome suggestions.]

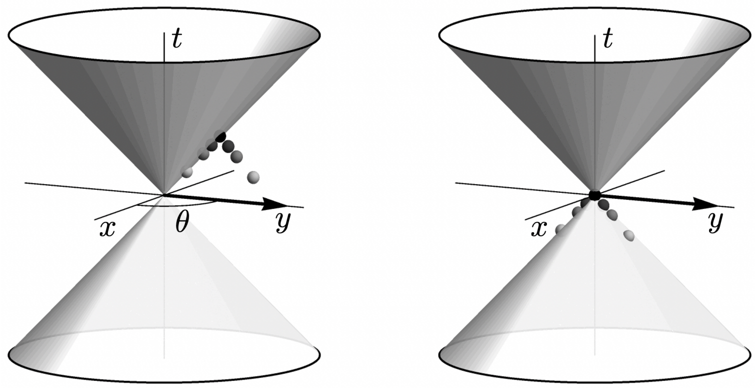

Figure 6: The spatiotemporally local form of the three dimensional wave equation (40) asks us to compare, for each direction picked out by a set of angles θ and ϕ, (a) how the values of the function u are changing as you approach a point displaced from the point of interest by an arbitrarily amount along that direction into the future light-cone from that direction along the past light-cone and the opposite direction to (b) how the values of u are changing as you approach the point of interest from that direction along the past light-cone and the opposite direction. This is shown in the two images above depicting only the x, y, and t dimensions. The angle θ picks out the y direction. The darkening dots show different points where u would be evaluated in calculating the light-cone derivatives for this direction.

where the ϕ that appears here is the scalar potential, not to be confused with the azimuthal angle. Here the density of charge ρ acts as the source term. Sometimes ρ is thought of as a coarse-grained blurring of an underlying spiky charge distribution of distinct bodies. Because it is a kind of average of the charge over a small region, you might worry that it involves looking outside of the past light-cone—and thus that (42) may fail to be a spatiotemporally local law. Let us think of ρ as (at least within the context of the theory) fundamental, either including delta functions (if matter is modeled as point charges) or smoothly varying (if matter is modeled using charge distributions). Then, we can take the value of ρ at a point to be a space-time point property and (42) fits the mold of a spatiotemporally local dynamical law.

The wave equation for the vector potential (19) is three separate three dimensional wave equations of the earlier form (30), one for each component of the potential. We can formulate it as

$$\int \frac{d\Omega}{4\pi}\left(\left(\frac{\partial}{\partial\eta}\right)^{f}\left[\left(\frac{\partial}{\partial\eta}\right)^{p}+2\left(\frac{\partial}{\partial\xi}\right)^{p}\right]\right)\vec{A}=-\frac{\pi}{c}\vec{J}\,. \tag{43}$$

For point charges, current density $\vec{J}$ that sources the vector potential can be formed by combining contributions of the form $\rho\vec{v}$ associated with each point charge, where ρ is a delta function centered on the charge's location and $\vec{v}$ is the charge's velocity (Landau & Lifshitz, 1971, sec. 28). As before, the velocity can be understood as a past derivative of the body's location and thus depends on an arbitrarily short segment of its past trajectory. If the body has never moved faster than the speed of light, then this trajectory will lie within the past light-cone and the velocity will be a past light-cone neighborhood property. The current density will thus also be a past light-cone neighborhood property. What if charged matter is modeled instead by a charge distribution? In that case, the current density at a moment cannot be read off from the present charge distribution and its recent history. Maudlin (2018, pg. 8) gives the example of a uniform sphere that maintains a uniform charge density. This is compatible with there being no current density or with their being a current density corresponding to a rotation of the charge about some axis. Thus, unlike the velocity of a point charge, the current density of a continuum cannot be reduced to the historic behavior of charge. One option would be to treat the charge distribution and current density as distinct properties of the instantaneous state. The current density then becomes a space-time point property, not a past light-cone neighborhood property. Again, (43) counts as a spatiotemporally local dynamical law.

In this paper, we have assumed that velocity should be reduced to changes in position over time. There is an alternative view available according to which instantaneous velocity is a genuine feature of the world at a moment that ought not be reduced to changes in position. Lange (2005) calls this view "velocity primitivism." As he puts it, "instantaneous velocity and trajectory are related only by virtue of natural law, not by metaphysical necessity." For the current density of a continuum, we have ended up in a similar position. Current density and charge density will be related by a law of nature: the continuity equation for charge,

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J} \ . \tag{44}$$

This equation can be derived from Maxwell's equations and should be derivable from the fundamental laws governing the evolution of the charged matter.[24] Thus, the continuity equation for charge might be regarded as a non-fundamental law (that, because of the divergence, is not spatiotemporally local—which is fine for a non-fundamental law).

We have been attempting to fit electromagnetism into the relativistic time evolution paradigm and thus far we have focused on the laws governing the evolution of the electromagnetic field. To complete the task, we must also analyze the forces on charged matter and the reaction of matter to these forces. First, let us consider the Lorentz force law for point charges (15). Using the potentials, we can rewrite this law as[25]

$$\begin{aligned}
\vec{F} &= -q\vec{\nabla}\phi - \frac{q}{c}\frac{\partial \vec{A}}{\partial t} + \frac{q}{c}\vec{v} \times \left(\vec{\nabla} \times \vec{A}\right) \\
&= -q\vec{\nabla}\phi - \frac{q}{c}\frac{\partial \vec{A}}{\partial t} + \frac{q}{c}\left(\vec{\nabla}\left(\vec{v} \cdot \vec{A}\right) - \left(\vec{v} \cdot \vec{\nabla}\right)\vec{A}\right) \\
&= -q\vec{\nabla}\left(\phi - \frac{1}{c}\vec{v} \cdot \vec{A}\right) - \frac{q}{c}\frac{D\vec{A}}{Dt} \ .
\end{aligned} \tag{45}$$

In the last line, $\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{v} \cdot \vec{\nabla})$ is the convective derivative, a derivative taken along the path of the charge. If we take this to be a past derivative and assume that the particle has

[24] For example, if matter is modeled as a classical Klein-Gordon or Dirac field then conservation of charge can be derived from the wave equations governing these fields.

[25] See Landau & Lifshitz (1971, sec. 17); Semon & Taylor (1996, pg. 1365); Griffiths (2013, pg. 442).

not moved faster than the speed of light, then the final term is a past light-cone neighborhood property. The remainder of the expression is a gradient that appears to require looking outside the past light-cone, and thus to cause trouble if we want the force to be a past light-cone neighborhood property so that the dynamics are spatiotemporally local. There is a way to address this problem, though it might feel like a cheap trick. In analogy with (25), we can introduce coordinates $\eta_i = x_i + ct$ and $\xi_i = x_i - ct$ to replace the gradient and write the x_i-th component of the force as

$$F_i = -q\left[\frac{\partial}{\partial\eta_i} + \frac{\partial}{\partial\xi_i}\right]\left(\phi - \frac{1}{c}\vec{v}\cdot\vec{A}\right) - \frac{q}{c}\frac{DA_i}{Dt} \ , \tag{46}$$

where i is an index on the spatial dimensions and should not be confused with the earlier use of i as an index on the charges. Making all of the derivatives past derivatives gives the force law

$$F_i = -q\left[\left(\frac{\partial}{\partial\eta_i}\right)^p + \left(\frac{\partial}{\partial\xi_i}\right)^p\right]\left(\phi - \frac{1}{c}\vec{v}^{\,p}\cdot\vec{A}\right) - \frac{q}{c}\left(\frac{D}{Dt}\right)^p A_i \ . \tag{47}$$

To clarify that the derivatives do not act on $\vec{v}^{\,p}$, this can be rewritten as

$$F_i = -q\left[\left(\frac{\partial}{\partial\eta_i}\right)^p + \left(\frac{\partial}{\partial\xi_i}\right)^p\right]\phi + \frac{q}{c}\vec{v}^{\,p}\cdot\left(\left[\left(\frac{\partial}{\partial\eta_i}\right)^p + \left(\frac{\partial}{\partial\xi_i}\right)^p\right]\vec{A}\right) - \frac{q}{c}\left(\frac{D}{Dt}\right)^p A_i \ . \tag{48}$$

In words: the force from the electromagnetic field in a given direction depends on (i) the way the scalar potential is changing forward and backward in that direction along the past light-cone, (ii) the dot product of the charge's velocity with the way the vector potential is changing forward and backward in that direction along the past light-cone, and (iii) the time derivative of the vector potential along the past trajectory of the charge.

Modeling matter as a continuous charge distribution, the above Lorentz force law for point charges (48) becomes an equation for the force density f_i in the x_i-th direction,

$$f_i = -q\left[\left(\frac{\partial}{\partial\eta_i}\right)^p + \left(\frac{\partial}{\partial\xi_i}\right)^p\right]\phi + \frac{q}{c}\vec{v}\cdot\left(\left[\left(\frac{\partial}{\partial\eta_i}\right)^p + \left(\frac{\partial}{\partial\xi_i}\right)^p\right]\vec{A}\right) - \frac{q}{c}\left(\frac{D}{Dt}\right)^p A_i \ , \tag{49}$$

where the velocity that appears here is a function of space and time.

As the final piece of the puzzle, let us now discuss the way that charges respond to forces. That is not always considered part of electromagnetism proper and does not appear in (11)–(15), but it would be needed to arrive at a complete theory that has a hope of fitting the relativistic time evolution paradigm. For point charges, we might use Newton's second law (4), $\vec{F} = m\vec{a}^{\,pf}$, as a non-relativistic approximation to the reaction of charges to forces. The relativistic law would be

$$\vec{F} = \left(\frac{d}{dt}\right)^f \vec{p}^{\,p} \ , \tag{50}$$

using a relativistic expression for the momentum,

$$\vec{p}^{\,p} = \frac{m\vec{v}^{\,p}}{\sqrt{1 - \frac{|\vec{v}^{\,p}|^2}{c^2}}} \ . \tag{51}$$

From (50) and (51) you can see that no matter how much force you exert on a massive body to increase its momentum, you cannot accelerate it beyond the speed of light—the momentum approaches infinity as the speed approaches c. Because the particles do not move faster than c, the momentum $\vec{p}^p$ will be a past light-cone neighborhood property and the force $\vec{F}$ will be a past-to-future light-cone neighborhood property. As in section 4, you could combine the non-dynamical Lorentz force law (47) and the dynamical reaction law (50) into a single dynamical law (assuming that the only forces at play are electromagnetic forces).

If the matter is modeled using continuous distributions of charge instead of point charges, the reaction of matter to forces is more complicated and will depend on the nature of the matter. You would need to couple some theory of the matter with the above laws of electromagnetism to get a complete theory that could fit the relativistic time evolution paradigm. A neatly relativistic way to do this would be to model the matter using a classical Klein-Gordon or Dirac field in a Maxwell-Klein-Gordon or Maxwell-Dirac classical field theory. Then, one would need to show that the wave equations for these fields can be formulated in ways that fit the relativistic time evolution paradigm, as we have seen that the wave equations of electromagnetism can—(42) and (43). For the Klein-Gordon field, that is straightforward as we can simply carry over the techniques that were used for the electromagnetic field.[26] For the Dirac field, it is not so clear how to formulate the wave equation as a spatiotemporally local dynamical law. Note that with either the Klein-Gordon or Dirac equation in place, the Lorentz force law (49) is not needed as a fundamental non-dynamical law (though it could still be regarded as a non-fundamental law giving the density of force exerted by the electromagnetic field on the Klein-Gordon or Dirac field[27]).

Putting it all together, we have arrived at a way of formulating electromagnetism that seems like it could fit the relativistic time evolution paradigm. For point charges, we can take the laws to be given by the wave equations for the potentials, (42) and (43), the Lorentz force law, (48), and the relativistic version of Newton's second law (50). However, there is danger on the horizon. This version of electromagnetism could run into problems of self-interaction like those faced by Newtonian gravity with point masses (discussed in section 4). Just as the gravitational field becomes infinite as you approach a point mass and ill-defined at its location, the electric field becomes infinite as you approach a point charge and ill-defined at its location. This causes trouble for the standard Lorentz force law (15). Does our new formulation of the force law (47) face the same problems? For a single point charge q at rest, we can use the scalar potential $\phi = \frac{q}{r}$ and set the vector potential to zero. The force law (15) yields

$$F_i = -q\left[\left(\frac{\partial}{\partial \eta_i}\right)^p + \left(\frac{\partial}{\partial \xi_i}\right)^p\right]\left(\frac{q}{r}\right) \ . \tag{52}$$

Focusing on the x_1 direction, the first derivative approaches from the left (along the past light-cone) and the second approaches from the right (along the past light-cone). The two

[26] You can prove the same kind of causality theorem for the Klein-Gordon equation as the one we discussed for the wave equation near the beginning of this section Wald (1984, sec. 10.1); Strauss (2008, pg. 234, problem 8).

[27] The idea that fields can experience forces is discussed in Sebens (2018, 2021).

contributions cancel, as they are both infinite but with opposite signs.[28] There is no force on the charge from its own electromagnetic field. That is nice. More work could be done to see exactly how well this formulation of electromagnetism can handle self-interaction (and whether it can recover effects like radiation reaction), but it is interesting to see that it has a way to solve the simplest problem of self-interaction. Perhaps this will suffice for addressing the problems of self-interaction or perhaps we will have to avail ourselves of one of the strategies that have been pursued for solving the problems of self-interaction for point charges in electromagnetism, such as replacing the Lorentz force law with the Lorentz-Dirac force law.[29]

We have seen that this formulation of electromagnetism with point charges satisfies the first two conditions of the relativistic time evolution paradigm: the dynamical and non-dynamical laws take the correct form. We have not yet settled whether the formulation satisfies the third condition as we have not yet settled whether the theory is deterministic. To do so, we must figure out whether, in general, an arbitrarily thin law-abiding time slice yields a unique law-abiding future. That requires looking at the coupled equations for the field and particle dynamics—not merely finding the field produced by a specified source or the force from a specified field.[30] Doing this honestly would require a solution to the aforementioned problems of self-interaction. There is another complication as well. Hartenstein & Hubert (2021) have shown that specifying law-abiding combinations of electromagnetic field and point charge states at an instant generically yields infinities or discontinuities that propagate along the future light-cone (called "shock fronts") and lead to ill-defined dynamics.[31] These shock fronts occur because the arbitrarily chosen field around a point charge does not capture the way that the charge's past motion would have acted as a source for the field. For example, at some moment you might have a particle with an initial velocity surrounded by the Coulomb field of a stationary point charge. That is not compatible with any reasonable past. Requiring that the input to the laws be an (arbitrarily short) time interval where the laws are obeyed should help in addressing these problems because we are assuming that the interaction between particle and field has been well-behaved in the immediate past of the moment after which we are calculating the future evolution.

Shifting now to consider electromagnetism with a continuous charge distribution, the wave equations for the potentials, (42) and (43), would again be laws. In addition, we would need law(s) determining the evolution of the charged matter. Although the Lorentz force density law, (49), can be formulated as a spatiotemporally local dynamical law, it will not be a fundamental law for certain classical completions of electromagnetism. In particular, if we model the evolution of matter using the Klein-Gordon or Dirac equations in Maxwell-Klein-Gordon or Maxwell-Dirac field theory, it is not needed. As was discussed earlier, the Klein-Gordon equation can be

[28] If we take the value of ϕ at the particle's location to be positive or negative infinity (depending on the sign of q), then the two derivatives are infinite with opposite signs. If we take the value of ϕ at the particle's location to be ill-defined, then the derivatives are ill-defined as well (unless we use Easwaran's open-ended derivatives from section 2).

[29] See Frisch (2005, pg. 59–63); Earman (2011, sec. 3); Kiessling (2011); Lazarovici (2018, sec. 3.1).

[30] See Frisch (2004, sec. 2); Frisch (2005, pg. 32–35).

[31] See also Lazarovici (2018, sec. 8.1).

formulated as a spatiotemporally local dynamical law and one might find a way to do so for the Dirac equation. The Maxwell-Klein-Gordon and Maxwell-Dirac field theories could then satisfy the first two conditions of the relativistic time evolution paradigm. They may satisfy the third condition as well—the dynamics appears to be deterministic and, although self-interaction is present, there is no infinite or ill-defined self-interaction.[32]

In this section we have seen that electromagnetism in the Lorenz gauge can be formulated in a way that satisfies the first two conditions of the relativistic time evolution paradigm, with the dynamical and non-dynamical laws taking an appropriate form. For point charges, the laws would be (42), (43), (48), and (50). These laws are serviceable, but they are not the most elegant and it would be valuable to search for a better spatiotemporally local formulation of electromagnetism.

The third condition of the relativistic time evolution paradigm is complicated by problems of self-interaction, but determinism can potentially be achieved if we reject point charges and instead treat the charged matter as a continuous charge distribution (as is done in Maxwell-Klein-Gordon or Maxwell-Dirac field theory).

8 Conclusion

This article opened with a quote from Maudlin presenting the idea of dynamic production, a quote that comes from the end of the final chapter of his book *The Metaphysics Within Physics*. Here is that quote again, this time with more context:

> "The non-Humean [dynamic production] package is, I think, much closer to the intuitive picture of the world that we begin our investigations with. Certainly, the fundamental asymmetry in the passage of time is inherent in our basic initial conception of the world, and the fundamental status of the laws of physics is, I think, implicit in physical practice. Both of the strands of our initial picture of the world weave together in the notion of a productive explanation, or account, of the physical universe itself. The universe, as well as all the smaller parts of it, is *made*: it is an ongoing enterprise, generated from a beginning and guided towards its future by physical law. ... I don't think that *scientific results* have, as yet, impeached the basic non-Humean picture, and no *philosophical arguments* give us reason to displace it. The metaphysics within physics is, as of now, non-Humean, and we can do no better as philosophers than embrace it." (Maudlin, 2007, pg. 182–183)

The appeal of the dynamic production account of laws is that it seems to be such a natural way of understanding much of physics. We have seen here that, even for the parts of physics that seem most conducive to an interpretation in terms of dynamic production (classical mechanics and electromagnetism), there are serious challenges that arise. We have also seen that these challenges can be addressed. Being careful about the inputs and outputs, we can understand

[32] See Sebens (2023).

how the laws produce the future and how that production might be incorporated in a relativistic picture of reality. Let us now turn briefly to two physical theories that are more hostile to an interpretation in terms of dynamic production: quantum field theory and general relativity. The purpose of this quick treatment is just to suggest that the dynamic production account has a shot at being extended to our most successful physical theories, and should not be viewed as roadblocked in a way that would make the kind of project pursued here a futile exercise.

For the non-relativistic quantum mechanics of a fixed number of particles, there exist a handful of "interpretations of quantum mechanics" that give explicit proposals about the laws and ontology—such as GRW theory, Bohmian mechanics, the many-worlds interpretation, and the many interacting worlds approach. These four options can all be fit straightforwardly into the time evolution paradigm, with either first-order or second-order dynamics depending on the interpretation. The many-worlds interpretation simply takes the first-order Schrödinger equation to give the dynamics of the wave function. This is naturally interpreted as taking the wave function at a moment as input (with no need to look to the arbitrarily-short past) and returning as output the way the wave function changes into the arbitrarily short future (Easwaran, 2014, pg. 857). GRW Theory introduces stochastic exceptions to the Schrödinger equation that can be understood as giving probabilities for different behavior in the arbitrarily short future (Sebens, 2015a, sec. 2). Bohmian mechanics is often presented as adding a first-order guidance equation that is another law, beyond the Schrödinger equation, determining the velocities of particles from their locations and the wave function (velocities that could be interpreted as future neighborhood properties). However, a second-order guidance equation can be used instead provided we impose the first-order guidance equation as a restriction on initial conditions (Goldstein & Struyve, 2015; Dewdney, 2023). In the many interacting worlds approach, the dynamics is second-order as particles are reacting to forces by Newton's second law and (at the fundamental level) there is no wave function evolving by the Schrödinger equation (Hall *et al.*, 2014; Sebens, 2015b). The fact that quantum theories normally posit first-order dynamics has been used by Builes & Impagnatiello (forthcoming) to argue that we live in a world where the present moment is enough to produce the future (the arbitrarily-short past is not needed as input)—in their words, "our universe is Markovian." I prefer to leave open whether the fundamental dynamics will turn out to be first-order or second-order.

Deciding between these competing interpretations of quantum mechanics is difficult. The many-worlds interpretation has been touted as "the only game in town" because it is the only option that can be immediately extended to relativistic quantum field theory (Wallace, 2012, pg. 35; Wallace, 2023). That virtue could be challenged either because the many-worlds interpretation has other serious problems (Adlam, 2023) or because it is not so easy to formulate quantum field theory in terms of a quantum state evolving by a Schrödinger equation (Sebens, 2022a).[33] The problem for Schrödinger evolution is that it is unclear whether the quantum state is a particle wave function (in Fock space) or a field wave functional, though hopefully one of these proposals can be made to work. Although the other interpretations may not be as straightforward to extend to quantum field theory as the many-worlds interpretation, there exist a number of promising strategies for doing so. The attempts to extend GRW and Bohmian mechanics to quantum field theory include non-local interactions that are incompatible with the relativistic time evolution paradigm (Maudlin, 2019, ch. 7). This is to be expected, as it is the lesson of EPR and Bell's theorem (along with the relevant experimental tests) that any single-world quantum theory must include such non-local interactions (Maudlin, 2014, 2019). One might react to this situation by abandoning the relativistic time evolution paradigm and instead simply seeking theories that fit the original time evolution paradigm, accepting that dynamic production occurs relative to some preferred foliation. Alternatively, one might work to show that a theory with multiple worlds (like the many-worlds interpretation or the many interacting worlds approach) can be cast in a form that fits the relativistic time evolution paradigm. Of course, one could also react to this solution by abandoning dynamic production (Adlam, 2023). I simply want to highlight that there are ways to potentially retain dynamic production in quantum field theory.

Let us now move on to general relativity. In the introduction to their paper, Chen & Goldstein (2022) present the "Einstein equation (of general relativity)" (a.k.a. the Einstein field equations) as one of the motivations for adopting their laws-as-constraints account, over a dynamic production account, because it is an equation that "in its usual presentation is non-dynamical." Later, they return to the issue and explain that "There are ways of converting [the Einstein field equations] into FLOTEs [fundamental laws of temporal evolution] that are suitable for a dynamic productive interpretation." Chen & Goldstein (2022) give the ADM formalism as an example of a way this might be done (Arnowitt *et al.*, 1962), complaining that this formalism discard solutions that are not globally hyperbolic (a bug that might be regarded as a feature if our universe is indeed globally hyperbolic and this formalism more precisely narrows down the space of physical possibilities[34]). Along similar lines, Adlam (2022a, sec. 2.2) writes

> "...there are a number of theories in modern physics where it doesn't make a great deal of sense to have a rigid division between 'state space' and 'evolution laws.' For example, a solution to the Einstein equations of General Relativity is not a state at a time but an entire history of a universe ... so it doesn't seem to require any concept of time evolution at all. A time-evolution formulation of the Einstein equations does exist [(Ringström, 2009; Fourès-Bruhat, 1952)], but the original global formulation remains central to research in the field and there seems no obvious reason to think that the time-evolution formulation must be more fundamental."

[33] Adlam (2022a, sec. 2.1) takes the use of path integrals in quantum field theory to suggest that the theory might best be understood outside the time evolution paradigm. We discussed path integrals in classical mechanics at the end of section 4 and I would say the same thing here. While it may be possible to formulate the theory outside the time evolution paradigm, path integrals in quantum field theory can be viewed as tools for calculating state evolution.

[34] See Maudlin (2007, pg. 175, 189–191).

For our purposes here, the most important thing to note is that both Chen & Goldstein (2022) and Adlam (2022a) acknowledge that there are versions of general relativity that seem to broadly fit the time evolution paradigm. So, although there remains work to be done for a proponent of dynamic production, there is a path forward. One version of general relativity that seems especially well-suited to an interpretation in terms of dynamic production is the classical spin-2 field theory approach (Salimkhani, 2020; Linnemann *et al.*, forthcoming). I have elsewhere called this the field-theoretic approach and contrasted it with the prevailing geometric approach (Sebens, 2022b, sec. 5). On the field-theoretic approach, gravity is a field on flat Minkowski spacetime much like the electromagnetic field and the field equations could thus potentially be fit into the relativistic time evolution paradigm. Because the field-theoretic approach treats gravity as similar to other physical interactions, it is arguably an attractive starting point for theories of quantum gravity.

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